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Examining 8th grade students' van Hiele geometry thinking levels, their proof writing and justification skills

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Abstract

The aim of this study is to examine 8th grade students' Van Hiele geometry thinking levels, proof writing and justification skills. The research was carried out with sixteen eighth grade students of a private school in Sivas. Participants were selected based on an easily accessible sample. The data were collected from the students who participated in the face-to-face teaching process in their schools and volunteered to participate in the research on the data collection day in the fall semester of the 2020-2021 academic year. The data collection tool is the Van Hile geometry test and the geometry proofing test. The document analysis method was adopted in the study. While analyzing geometry proof writing and justification test data, geometry proof writing test evaluation rubric and justification evaluation rubric were used. As a result of the analysis of the data obtained, there was a linear relationship between the students' Van Hiele thinking levels and their proof writing skills, while there was no positive relationship between their justification skills and their Van Hiele thinking levels and their proof writing skills.

Keywords: Van Hiele geometry thinking level, proof, justification

1. Introduction

The purpose of geometry education is to help us make sense of the world we live in. Therefore, this aim has been tried to be achieved since pre-school. Teaching geometry should contribute to the development of students' ability to visualize, think critically, reduce the objects they encounter in daily life to two dimensions, solve problems, make assumptions, make logical inferences and prove. The first study on geometric thinking levels was made by the Van Hiele couple. The Van Hiele model mentions the existence of five levels of thinking: visualization (level 0) analysis (level 1), informal deduction (level 2), formal deduction (level 3), rigor (level 4) (Usiskin, 1982). Students pass the levels in sequential order. However, not all students can pass the levels at the same rate. In other

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words, Van Hiele levels may be different in students in the same class (Senk 1989). On the basis of geometry teaching, students are expected to have reasoning and justification skills (MEB, 2020). In geometric reasoning and justification skills; geometric thinking, making generalizations, and constructing geometric ideas in a meaningful way. Therefore, it is seen that there is a direct relationship between the levels of geometric thinking and the skills of proving and justifying (Driscoll, DiMatteo, Nikula & Egan, 2007). Therefore, in this study, Van Hiele geometry thinking levels, proof writing and justification skills of 8^{th} grade students were examined. The conceptual framework of the study, proof, justification and Van Hiele geometric thinking levels are explained below.

1.1. Van Hiele Levels of Geometric Thinking

Van Hiele geometric thinking level is a model that explains how geometry is learned and how students perceive geometric concepts. While learning geometry in the model, students are expected to go through stages called visual analysis, simple inference, formal inference and systematic thinking. According to Van Hiele, the student goes through the following five levels supported by appropriate teaching experiences, and the student cannot reach another level of thinking without going through the previous levels.

Level I (Visualization): Students at this level can learn the names of shapes and recognize them as a whole. Pesen (2006) stated that students at this level could not comprehend geometric shapes by using their definitions, they also benefited from daily life examples according to their observations around them, they were compared and named. Students at this level recognize geometric shapes. At this level, students are expected to name the given shapes and choose the desired shape among other shapes (Baki, 2008).

Level II (Analysis): At this level, students analyze the properties of geometric objects and can express their properties. A student at this level can understand that shapes have more than their appearance, with their features, and can understand that the features of the shape also represent the features of the class to which the shape belongs. Thus, they can fully evaluate the properties of the shapes together. From here, he can generalize certain features to the whole. At this level, students are expected to answer the questions of "determine the properties of given shapes and express them" (Baki, 2008).

Level III (Informal Deduction): The difference of this level from the first two levels is that students begin to understand the relationship between objects and shapes. Students make sense of definitions and axioms, but logical conclusions are not understood at this level. At this level, students can logically realize the relationships between shapes, but cannot work in a mathematical system. They can follow the proof but cannot write proof. Since students at this level can see the relationship between objects and shapes, they can express that each cube is a square prism at the same time, and every square prism is also a rectangular prism, in this case the cube is also a rectangular prism. The most important feature of this level is the ability to make connections between objects and shapes. At this level, students are expected to define the given geometric situation, find the relationships between the properties of the given shapes, and determine the necessary and sufficient conditions for the given proof (Baki, 2008).

Level IV (Formal Deduction): The most important feature of this level is that students come to the level of being able to write proofs. Students use the axioms and theorems they know when writing proofs. At this level, students can understand the importance of proof based on assumptions and theorems and can make proofs. It could use an axiomatic setup. The properties of objects and shapes become independent of each other. At this level, students are expected to answer the questions "Make the given proof and support it with logical inferences" (Baki, 2008).

Level V (Rigor): This is the last advanced level of Van Hiele's theory of geometric thinking. According to Hoffer (1981), students at this level can understand the differences between different axiomatic systems. They can also make sense of the definitions, theorems and axioms of Euclidean geometry in non-Euclidean geometries. They can understand the differences of axiomatic systems and recognize relationships. This can be seen as an area where axiomatic systems can work (Karapınar, 2018). At this level, the students were asked “What is the sum of the interior angles of an equilateral triangle drawn on a sphere?” and “What shape does the square that is tried to be drawn on the sphere turn into?” (Baki, 2008) are expected to answer the questions.

One of the main features of this learning model is that it is gradual. According to Van Hiele geometric thinking levels, students pass the levels in a sequential order while learning geometry. In order for a student to be at a certain level, they must have passed the previous levels. The transition in levels is not dependent on age. It depends on their level of learning and understanding of geometry. In other words, a primary school student can be at the third level, and a university student can be at the second level.

1.2. Proof in Mathematics Education

In its most general sense, proof is the demonstration that a statement is true or false. Proof is an activity in which the student can reveal his reasoning ability (Zaimoğlu, 2012). Different reasoning enables knowledge to be formed in different directions (Altıparmak & Öziş, 2005). In this context, one of the special aims of the mathematics curriculum is to be able to easily express their own thoughts and reasoning in the problem solving process, as well as to see the deficiencies or gaps in the mathematical reasoning of others (MEB, 2020). For this reason, the place of proof in mathematics education is important. It is possible to talk about many important functions of proof in mathematics education. The most basic role of proof is to reveal the truth of a claim. Students generally understand and experience the concept of proof with this dimension. In addition, it is a fact that the routines related to the procedures and proof methods are brought to attention rather than the logic used in the proof process within the scope of school mathematics. It is stated that such situations cause the students' knowledge and thoughts about proof to be restricted to routines. For many students, the proofs do not mean anything beyond being rather pointless exercises with routine practices, carried out by following the methods and procedures suggested by the teachers. According to mathematics educators, the main utility of proof is related to its role in illuminating why and why a claim is true. It is this feature of the proof that supports the development of thought in students. For this reason, educators evaluate proofs in two basic categories according to their qualifications. First, they are proofs that superficially show the truth of a claim without too much questioning; the second is explanatory proofs that reveal the truth of that claim in all its depth in a way that will answer the questions of why and why (Bayazıt, 2017).

1.3. Justification Skill

At the core of the proof are justifications. It is stated that the importance of proof and justification in primary and secondary school mathematics is very important not only for doing mathematics but also for learning mathematics (Stylianides, 2006).
In the literature, there are different views for the definition of justification. Justification is defined as the reasons presented to convince oneself of an event or situation for which a decision has been made, or the person's having sufficient evidence to confirm his claim (Akkan, Öztürk & Akkan, 2017).

Unless students' reasoning skills are developed, mathematics remains for students only as a collection of calculations and drawings performed by following certain rules and without thinking about what they are (Ross, 1998).

Many studies have shown that students do not have difficulty in performing the rules and operations in mathematics, but they do not know the underlying meanings of the operations and mathematical ideas they do (Hadas, Hershkovittz, & Schwarz, 2000).Considering the studies on Van Hiele geometry thinking levels at secondary school level, they concluded that the geometric thinking levels of secondary school students were not at the desired level (Alaylı, 2012; Fidan & Türnüklü, 2010; Idris, 2009; Senk, 1989; Usiskin, 1982). In studies on secondary school students' proof writing skills, it has been observed that students' proof writing skills do not develop (Albayrak Bahtiyari, 2010; Zaimoğlu, 2012). Likewise, studies on the reasoning skills of secondary school students have also shown that their levels are low (Özmusul, 2018; Yackel, 2001; Arslan, 2007). When the literature is examined, there are two studies examining Van Hiele geometric thinking levels and proof writing skills at secondary school level. One of these studies was done by Senk (1989) and the other by Coşkun (2009). The results of two studies showed that students with high level of Van Hiele geometric thinking also had high proof writing skills. As a result, when the studies were examined, many studies were found on Van Hiele thinking levels, proof writing and justification skills at the secondary school level. But since there is no study examining the dual relationship or the relationship between these three, namely Van Hiele thinking levels and the skills of writing proof and justification, as in the study of Senk (1989) and Coşkun (2009), it is thought that this study will contribute to the literature. For this reason, in this study, 8th grade students' Van Hiele thinking levels and their ability to write geometric proofs and justification were examined in the context of the sum of the interior angles of the triangle. Answers were sought to the following questions.

1) What is the Van Hiele geometry thinking level of 8th grade students?

2) At what level are 8th grade students' ability to write proofs?

3) What is the level of reasoning skills of 8th grade students?

4) How are 8th grade students' Van Hiele geometry thinking levels, proof writing skills and justification skills?

2. Method

This study was carried out by document analysis method. The document analysis method is the systematic examination of existing documents or records as data sources, which includes the analysis of written sources containing information about the subjects to be researched. The document analysis method is used to reach the data for the purpose of the study and to determine the findings from these data (Çepni, 2010; Yıldırım & Şimşek, 2013).

2.1. Participant

The study group was determined according to the easily accessible sampling method, which is thought to allow the best explanation of the researched subject and to make the best contribution to the solution of the research problem. The participants of the study are 16 students who continue their education in the 8th grade of the 2020-2021 academic year in a private school located in the center of Sivas, participate in the face-to-face teaching process on the day the data will be collected and volunteer. Among these students, seven students are girls and nine students are boys. The names of the students participating in the study were named as S1, S2, S3….

2.2. Data Collection Tools

In order to examine the Van Hiele geometric thinking levels, proof writing and justification skills of 8th grade students, the students were asked the "Van Hiele geometry test" consisting of 25 questions, and the question "Show that the sum of the measures of the interior angles of a triangle is 180 degrees and explain the steps you took to convince your friends with the reasons". In order to determine the level of geometric thinking, the 'Van Hiele Geometry Test' developed by Usiskin (1982) was applied. This test was translated into Turkish by Baki in 1994 (Baki, 2008). The first five questions of the test are about visual shapes and aim to determine whether students can recognize geometric shapes by looking at the image of the shape. In the second group, five questions are about the properties of shapes. It aims to reveal whether the students recognize the shapes on the one hand, and whether they know the properties of the given shapes on the other hand. In the third group, five questions determine whether the students notice the relationships between the shapes. Now, the student who knows the questions in this group defines and proves that he has knowledge about axioms. The five questions in the fourth group are questions that can be reasoned and logically deduced. It can be determined whether the students are at the level of understanding and writing a proof from these questions. The last five questions of the test consist of questions that can determine whether the student can reason in Euclidean and non-Euclidean geometries. In order to determine the skills of writing proof and justification, show that the sum of the measures of the interior angles of the triangle is 180 degrees and explain the steps you took to convince your friends with the reasons. While they are expected to prove in the first part of the question, they are expected to write their reasons in the second part. The sum of the measures of the interior angles of a triangle is given in the 5th grade. Therefore, it was accepted that the 8^{th} grade students had the knowledge and skills related to this question. While asking this question, the students were not given a visual. It is also not expressed in mathematical language as show that $\overset{\Delta}{ABC}$ is a triangle and s(A) + s(B) + s(C) = 180°. The reason for this is to examine the steps and justifications of the students while finding the sum of the measures of the interior angles of a triangle, as well as the visuals they drew and the mathematical language they used.

2.3. Data Collection

The data were collected by the researcher from the 8^{th} grade students attending a face-to-face mathematics lesson in a private school located in the center of Sivas. The application of the two tests used in the study was carried out on the same day in two class hours with a 10-minute interval due to the pandemic conditions. The first lesson Van Hiele geometry test was applied to the students. The researcher only gave information about how the test should be done. In the second lesson, geometry proof test was applied. The researcher also told the students how they should follow this test. All 16 students participated in the study in these two courses.

2.4. Data analysis

2.4.1. Analysis of the Van Hiele geometry test

The Van Hiele geometry test consists of 25 questions. Each five-question section determines a level. According to Van Hiele, if at least three of the questions in each group are answered correctly, they are scored as zero if they give an incorrect answer. For example, if a student is at the second level, his score is 11000 (Baki, 2008). In this study, the criterion of correctly answering three of the five questions was used to assign the student to a level. In addition, in the level assignments, the scoring key developed by Usiskin (1982) was used in line with the answers given by the participants to the Van

Hiele geometry test. The scoring key determined for the Van Hiele geometry test is as follows:

- 1 score if it meets the criteria in questions 1-5
- 2 scores if it meets the criteria in questions 6-10
- 4 scores if it meets the criteria in questions 11-15
- 8 scores if it meets the criteria in questions between 16-20,
- If it meets the criteria in questions between 21-25, 16 scores are given. (Usiskin, 1982; 22).

2.4.2. Analysis of Proof Writing Test

Two rubrics were used while analyzing the proof writing test. The first one is the proof writing test evaluation rubric and the second one is the justification evaluation rubric. The purpose of preparing rubrics is to examine proof writing and justification skills more easily. While preparing the rubrics, expert opinions were taken from two mathematics educators. In addition, a pilot study was conducted. As a result of the opinions of the experts and the pilot study, it was seen that the rubrics were suitable for the study. While scoring according to the Proof Writing Test Evaluation Rubric. The proof writing evaluation rubric is given in Table 1.

Table 1. Proof Writing Test Evaluation Rubric

Criterion	**Score**
Situations where the problem is completely misunderstood, meaning that the problem is not understood or nothing is done.	0
Situations in which the problem is understood (Question was expressed verbally, the algebraic version of the problem was written or short notes were taken about this expression, graphs were drawn, tables were created, the expression/correctness of the given argument/proposition was tried with examples).	1
Situations in which what is asked in the question is understood (understands exactly what needs to be proved, determined the method of proof, created/realized the logical steps given for this, but could not fully conclude the proof or there are deficiencies/errors in some stages of the proof).	2
Situations where the proof is completed correctly.	3

As can be seen in the prepared evaluation rubric (Table 1), there are scores for each situation in the proof writing test. If correct proof is written, it gets 3 points, if wrong proof is written, it gets 0 points. Justification Evaluation Rubric is given in Table 2.

Table 2. Justification Evaluation Rubric

Code		Score
Fully justification	Answers that prove the question correctly and support it with correct mathematical justifications	3
Half meaningful justification	Proving the problem correctly, incomplete writing of the reason	2
	Incomplete proof of the problem, incomplete writing of the justification	
Faulty justification	Failure to write appropriate justification for the answer in questions where the proof is proved incorrectly, a transaction error is made, or conceptual errors are made.	1
No justification	The proof is not correct and the justification is not written.	0
	The proof is wrong and the justification is not written	
	Half of the proof and not writing the justification	

As it can be seen from Table 2, while evaluating the justifications of the students, if a correct justification is written in the question, 3 points are received, if not, 0 points are received.

3. Findings

In order to answer the sub-problems of the research, the students' Van Hiele geometry thinking levels, proof writing and justification scores are shown together in Table 3.

Table 3. Van Hiele geometry thinking levels, proof writing and justification scores

Student	Van Hiele Geometry Thinking level	Proof Writing score	Justification Score
S1	3	3	2
S2	1	3	0
S3	1	1	0
S4	2	1	0
S5	1	2	0
S6	1	0	0
S7	4	2	3
S8	2	1	0
S9	1	2	2
S10	3	1	0
S11	3	0	0
S12	1	1	0
S13	2	1	2
S14	3	1	0
S15	3	3	0
S16	3	1	0

According to the findings obtained from the Van Hiele geometric thinking test (as seen in Table 3), there are no students at the zero level. Six students (S2, S3, S5, S6, S9, S12) are at the first level, three students (S4, S8, S13) are at the second level, six students (S1, S10, S11, S14, S15, S16) are at the third level, and one student is at the third level. (S7) is at the fourth level. In other words, the majority of the students participating in the study are at the first and third level. According to the findings obtained from the proof writing test (as seen in Table 3), three students got three full points. Three students scored two points, eight students one point, two students zero points. Considering the justification scores, there is one student who got a full score. Three students got two points and 12 students got zero points. There is no student who gets a point. If geometric thinking levels, proof writing and justification scores are evaluated together, the student with the highest justification score is the student with the highest geometric thinking level (Level 4). It is seen that one of the students with a justification score of two is at the first level, while one is at the second level and the other is at the third level. S6 and S11 who got zero points from the question of writing proof are the students. S6's answer is given in Figure 1.

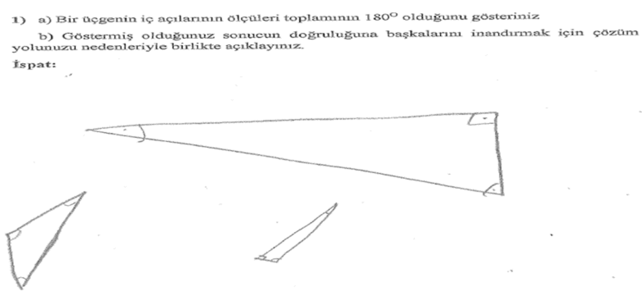


Figure 1. S6's answer

When Figure 1 is examined, three different triangles were drawn, no explanation was given, that is, no justification was made. For this reason, the justification score was given as a zero score. The students who got one point from the question of writing proof are S3, S4, S8, S10, S12, S13, S14 and S16. S12's answer is given in Figure 2.

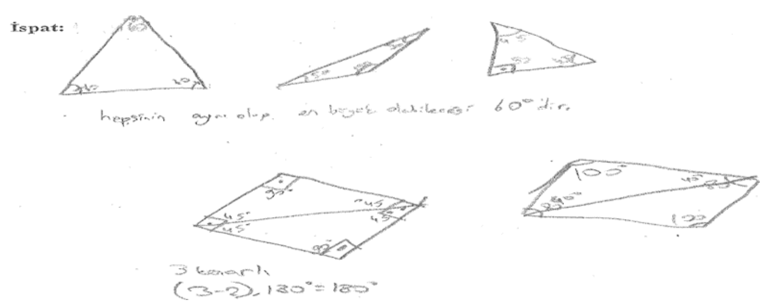


Figure 2. S12's answer

When the answer given in Figure 2 is examined, it is seen that S12 followed two different paths. In the first, he drew different triangles and gave different values to their interior angles and completed them to 1800. In the other, he drew different quadrilaterals and divided it into two. In fact, he understood the question, but could not fully prove it. A score is given according to the criteria of the proof rubric. The students who got two points from the question in writing proof are S5, S7 and S9. S7's answer is given in Figure 3.

Figure 3. S7's answer

As seen in Figure 3, T7 drew a triangle and made a right angle by combining the angles. However, he did not name the angles, so two points were given because it was not understood where and how the angle met while showing the right angle. The students who got three points from the question in writing proof are S1, S2 and S15. The answer of S1 is given in Figure 4.

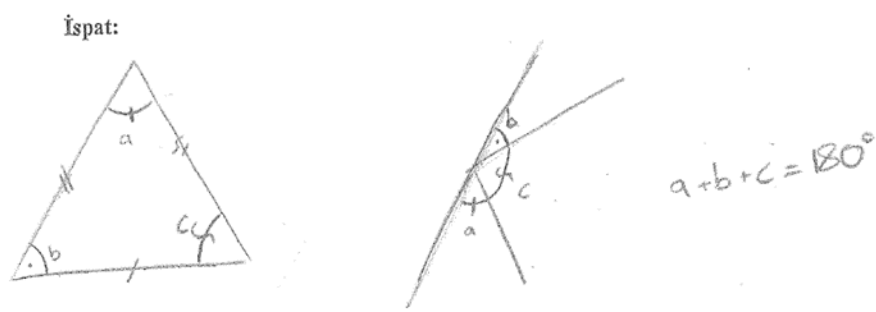
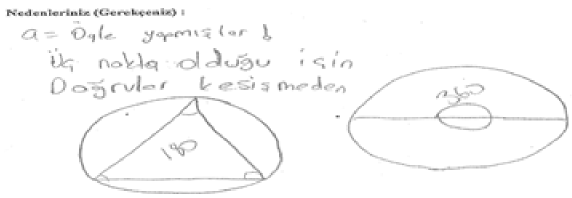


Figure 4. S1's answer

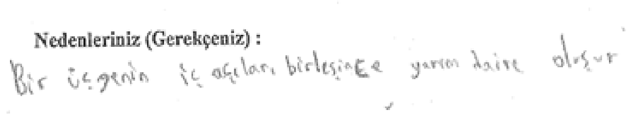
As shown in Figure 4, S1 drew the triangle and named the angles. He combined these angles and made a right angle. He received three points for fully proving the sum of the interior angles of a triangle. The students who got zero points without justification are S3, S4, S5, S6, S8, S10, S11, S12, S14, S15, S16. S6's answer to writing proof was given in figure 1. Here, too, what they wrote about the justifications is given in Figure 5.



Translation:
a=that's what they did!
because there are three points.
without the lines intersecting.

Figure 5. S6's justifications

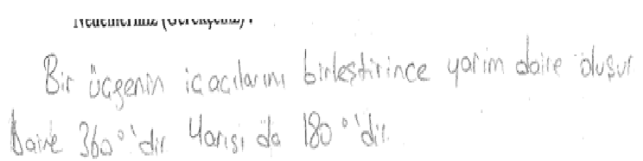
As seen in Figure 5, S6's justification is not compatible with his proof. In addition, he could not establish a connection with the fact that the sum of the interior angles of the triangle is 180°. That's why their justification was given zero points. There is no student who scored a point according to the justification skills rubric. Students who get two points according to the justification skills rubric are S9 and S13. S9's justification answer is given in Figure 6.



Translation:
When the interior angles of a triangle meet, a semicircle is formed.

Figure 6. S9's justifications

As can be seen in Figure 6, the justification is not clear. However, he should have done what the semi-circle formed when the interior angles of the triangle were combined. That's why two points are given. Students who get three points according to the justification skills rubric are S2 and S7. S7's answer for writing proof is given in Figure3. Its justification is given in Figure 7.



Translation:
When the interior angles of a triangle are joined, a semicircle is formed. The circle is 360 degrees. Half is 180 degrees

Figure 7. S7's justifications

As seen in Figure 7, he stated that when the interior angles of the triangle are joined, a semicircle is formed. He also stated that the circle is 360o and half of it is 180o and wrote his justification in accordance with his proof.

4. Discussion and Conclusion

When the Van Hiele geometric thinking levels of the students are examined, there is no student at the zero level. There are six students at the first level, three students at the second level, six students at the third level, and a fourth student. These findings are similar to the finding of Usiskin (1982), Senk (1989), Fidan and Türnüklü (2010) and Gül (2014), that the geometric thinking levels of secondary school students are not at the expected level. Gül (2014) stated that the Van Hiele geometric thinking levels of secondary school 8^{th} grade students are below the expected level (level II). Karakarçayıldız (2016) concluded that the geometric thinking levels of secondary school seventh grade students are low and the geometric thinking test scores of the students do not differ according to the variables of gender, pre-school education status, mother and father educational status. In Uzun (2019)'s study with 8^{th} graders, it was determined that students gathered mainly at the first level. It has been determined that 8^{th} grade students have low levels of Van Hiele geometric thinking, which is necessary for them to understand and interpret geometry. In order for students to be able to understand the geometry lesson, they must be at least at the second level (Senk, 1989). In this study, six students were seen at the first level. According to studies, these students' understanding and interpretation of geometry is low. It was observed that there were three students at the second level, six students at the third level, and one student at the fourth level. The Van Hiele geometric thinking levels of these ten students are at the required level.

In writing proof, three students got 3 full points. Three students scored two points, eight students one point, two students zero points. When the relevant literature is examined, Albayrak Bahtiari (2010) tried to describe the views of 8^{th} grade students on proof in mathematics education in his study. As a result, it has been determined that 8^{th} grade students have deficiencies in both proof and reasoning. Çalışkan (2012) examined the relationship between 8^{th} grade primary school students' mathematics achievement and their ability to prove, and found a positive relationship between students' ability to prove and their mathematics achievement. Zaimoğlu (2012), on the other hand, worked with 8^{th} grade students and examined their geometric proof and reasoning process in line with inductive and deductive reasoning. It has been revealed that the students do not fully understand the methods and techniques of proof.

Considering the justification scores, there is one student who got a full score. Three students got two points and 12 students got zero points. There is no student who gets a point. When the relevant literature is examined, Özmusul (2018) investigated the reasoning skills of 7th grade students in some of the geometry subjects. In this study, it was observed that the students' positive justification skills did not develop. Yackel (2001) stated that an acceptable mathematical explanation and justification in stealing can be counted as a socio-mathematical norm. He mentioned that giving reasons and explanations had a positive effect on the mathematical norm in the classroom. He also suggested that symbolic interaction emerges with explanation and justification. With symbolic interaction, students interact more with each other in the classroom and thus students try to make sense of each other's actions. In Arslan's (2007) study, the development of 6th, 7th and 8th grade students' reasoning and proving thinking was examined. In the study, it was observed that the level of justification of the students in general was low and it was determined that the students could not use the methods they should use sufficiently in this process.

When the students' Van Hiele geometry thinking levels, proof writing and justification scores are examined, one student's proof score is zero, two students one point, two students two points, and one student three points out of six students whose Van Hiele thinking level is at the first level. When we look at the justification scores, the score of five students is zero, and the score of one student is two. Among the students whose Van Hiele thinking level is at the second level, the proof scores of three students are one. Justification scores are zero. The proof scores of students whose Van Hiele level of thinking is at the third level are zero for one student, one for three students, and three for two students. Justification scores are zero for five students and two for one student. A student whose Van Hiele level of thinking is at the fourth level has a proof writing score of two. The justification score is three. When we look at the literature, there is no study that examines the relationship between these three elements. When we look at this study, it is seen that there is a relationship between students' Van Hiele geometry thinking levels and writing proofs, while their ability to justify is mostly lower than their Van Hiele thinking levels and ability to write proofs. This shows that students learn without knowing the reason for a given situation in mathematics and where it came from.

5. Recommendations

This study was conducted with 16 students who can be reached in a school that continues face-to-face education due to the pandemic. In future studies, the same study can be done with more participants. Quantitatively, the relationship between the scores can be looked at. Proof writing and justification skills can be examined in detail by conducting interviews. In addition, the relationship between academic achievement and proof writing and justification skills can be investigated.

Note: 1. This study was produced from the master thesis prepared by the second author under the supervision of the first author.
2. Within the scope of the research, ethics committee approval was obtained from the ethics committee of Sivas Cumhuriyet University with the decision dated 21.01.2021 and numbered 2021/26.
3. A part of this study was presented as an oral presentation at Gazi University International Congress of Turkish World Educational Sciences on November 22-24, 2021.

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