



Perspicuous technique in solving a square of a number through patterns and sequence: towards innovative teaching

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Abstract

This research paper investigated the effectiveness of the discovered formula in squaring numbers using patterns in a sequence introduced by the researchers as an innovation in teaching and learning mathematics. The researchers came up with the idea to use the patterns in sequence to develop a rule or formula for squaring numbers. The developed formula was evaluated as effective and efficient since the technique only used basic multiplication and addition that covers all numbers. Descriptive-developmental and evaluation research was utilized in this paper. Twenty professional teachers working in different capacities were purposively selected as respondents. Meanwhile, the technique was very effective as it promotes innovative and creative ease of introducing new methods to the learners with an overall weighted mean of 3.68. Thus, it increases students' engagement in mathematics. Likewise, the respondents, found the technique as timely and relevant, aids learners in squaring numbers without using a calculator, is engaging and enjoyable, provide opportunities to learners, and can be utilized in some mathematics competition. Therefore, this new pedagogical strategy will help students in squaring numbers easily. Lastly, it implies that teacher's methodology and strategy may affect the learning of students. Thus, innovations such as providing alternative solutions will increase their learning in mathematics.

Keywords: *Perspicuous Technique, Innovation, Innovative Teaching, Patterns and Sequence, Squaring of Numbers*

1. Introduction

1.1. Background of the Study

During our students' days until today as teachers teaching mathematics, it is very evident that interest and excitement to study mathematics play a vital role in the students' learning. Based on our observations, the students seemed to be interested when a teacher provides new things to them. Their curiosity arouses when they are eager to learn new things. Thus, as a math teachers, we tried to think to give new ways to learn mathematics, a way that is something creative and innovative. The success of learning mathematics is primarily determined by students' enjoyment of the subject. This, combined with other factors like a

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perceived utility in learning mathematics, will eventually boost students' desire to learn mathematics (Abdullahi, 2021).

Meanwhile, although not every child will grow up to be a senior mathematician, every child has the right to access learning opportunities at school to further his or her mathematical understanding. The way youngsters are taught mathematics has a direct impact on their mathematical success (Turgut, 2020). According to Alibraheim (2020), students' attitudes toward mathematics have a significant impact on their academic performance. However, worries about students' poor performance on mathematics tests have generated doubts about the reasons for their poor performance.

In the study conducted by Ossiannilsson (2018), he found that student engagement, retention, and academic results can all benefit from the use of innovative learning methodologies and learning spaces. It is critical that learning environments can adapt to future demands and continue to provide world-class teaching and learning opportunities through the design and implementation of cost-effective, adaptable, and future-based spaces.

Similarly, according to Kibria (2015) and Yeoh (2015) as cited by Subia (2018) about innovative teaching, higher educational institutions, through the teachers, should serve the long-term intellectual needs of students by providing new material which helps the student to gain new insights or open new channels of intellectual stimulation or enhance student's essential and creative thinking power. Teachers can achieve today's generation of educational needs for students through innovative teaching. However, research results showed that competencies in teachers for innovative teaching are lacking.

As a result, innovation in education can be defined as a novel pedagogical theory, methodological approach, instructional technique, teaching instrument, or theoretical structure that results in significant changes in teaching and learning and better student learning when applied to teaching and learning (Orhan, 2017).

Additionally, innovation is modifying or introducing new ideas is a strategy for economic progress that must be pursued. An individual's ability to create wealth is based on their ability to use readily available resources and apply fresh ideas and concepts to problem-solving tactics. Innovation is a method for facilitating value addition by combining creativity and practicability, as well as a strategy for solving problems that is purposeful and systematic. Innovation may gradually promote and drive education reform and advancement in a beneficial and favorable direction through thorough consideration, flexible decisions, and methodical use of fresh concepts and technologies (Amabile, 1996; Craft, 2005; Ritchhart, 2004; Rogers, 2003; Hung, 2017). Teaching is both science and art employing strategies, schemes, or tactics to achieve a goal. Techniques, to be practical, require the intelligence, maturity, and devotion of teachers. Furthermore, successful teachers should have the knowledge and a sense of purpose to rise above casual or conventional approaches and do things others cannot (Boiser, 2000 as cited by Subia, 2018).

Meanwhile, it is an arduous and critical move by the government to redesign the country's



primary education system. This move by the government with K to 12 program aims to cultivate highly-skilled graduates. The Department of Education (DepEd), led by Secretary Armin Luistro in 2011, made kindergarten a prerequisite to primary education.

This program changed the 10-year basic education curriculum to twelve years in grade school and six years in secondary, four years in junior high school, and two years in senior high school. This innovative action by the government puts the future of the students in a favorable situation. Innovation in various ways is needed to provide a new process of learning. Sometimes, teachers will not only depend on what is being written in a textbook. An innovative teacher should be creative in the delivery of instruction (Barlongo, 2015).

Squaring numbers is something fun and challenging for everyone. There are some techniques that everyone can use to get the square of a number quickly. However, these techniques are not yet established as a rule to be used in all cases. Squaring a number is multiplying a number by itself (Williams, 2021). On the other hand, some learners are having difficulty doing the algorithm way of multiplying two numbers.

But, there are several techniques that mathematics experts bring together to solve a square of a number. Subia (2006) introduced in his thesis the steps in solving shortcut techniques in squaring two-digit numbers ending in 5 using mental computation. Jackson (2019) on the other hand presented on his youtube channel the squares of all two-digit numbers using tricks such as squaring the individual numbers separately and adding the result to the product of the two-digit numbers. Other experts in squaring numbers use a grid, middle sum, place value, and math magic to perform the products (Gardner, 2011; Jensen, 2012).

Their contributions to the topic inspired the researchers to venture into other solutions in squaring numbers. Thus, the researchers made a lot of research and came up with the idea to use a sequence as an alternative formula for squaring two-digit numbers.

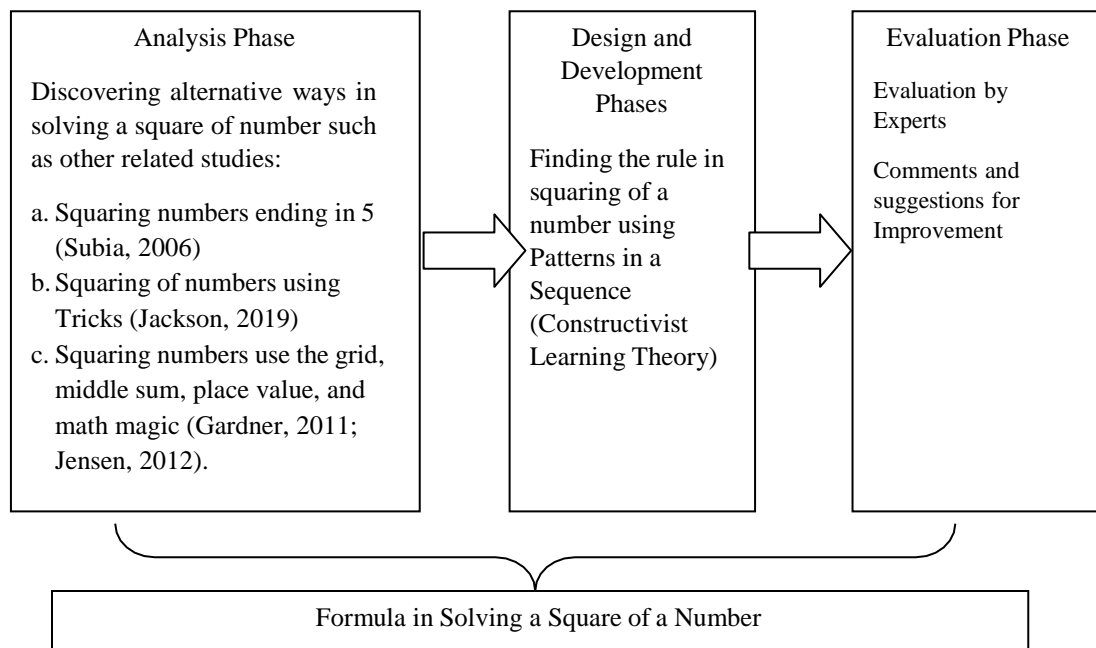
A sequence is a function whose domain is the set of positive integers. It also means an ordered list of numbers. In a sequence, each element is called a term, which corresponds to a natural number. Normally, a sequence has a rule or formula for the value of its terms. However, there are some sequences that the rule that cannot be easily determined so it requires a more in-depth examination of the given data. For that circumstance, we must find a recurrence relation. A recurrence relation defines the relationship between successive terms (Nivera & Lapinid, 2015).

As the researchers reflect, nowadays, teachers need to be creative and think of another way of presenting topics to learners. Mathematics is full of patterns; students and teachers need to investigate every angle of a problem. Innovative teaching is necessary for subjects such as Mathematics, where the students have negative perceptions towards it. The new ways of solving math will help them to be confident and overcome their fear of Mathematics. As teachers, we want our students to learn quickly; that's why a new technique is always captivating their interest to learn. In the different mathematics branches, squaring a number is one of the basic needs to solve a math problem.

In math, the researchers reflect on this. Complex problems and solutions make students feel anxious and not confident to learn mathematics. As mathematics teachers, it is necessary to help them understand the lessons by providing alternative solutions. This innovation makes us believe that interest follows once they feel it is easy to solve.

The scenarios, as mentioned, are the reasons to encourage teachers to be creative to satisfy the needs of the learners and the researcher to do a study about innovating pedagogical methods. This research will examine the new technique's effectiveness in squaring a number as innovation and alternative solutions. It also aims to encourage everyone to investigate by looking at the pattern to unlock new possibilities and make life easy. This research will help students solve the square of a number without using a calculator or any technological device and for teachers to captivate the learners' interest.

1.2. Conceptual Framework and Paradigm of the Study



The paradigm of the study showed the collection of processes and concepts needed in the operation of the study.

In taking a closer look at the paradigm, the study adopted the Phases of the Instructional Model (ADDE) of Developmental Research as redefined by Ibrahim (2016). Ibrahim redefined the stages of instructional design. In this research, there are only four phases: Analysis, Design, Development, and Evaluation, which are defined as how teachers and innovators can use the phases of conducting developmental research, the methodologies of data collection, and participants. Thus, this redefined model helps neophyte researchers with this kind of research.



The model started with the analyses of the ideas of other researchers in squaring of numbers. There are several techniques that mathematics experts bring together to solve a square of a number. Subia (2006) introduced in his thesis the steps in solving shortcut techniques in squaring two-digit numbers ending in 5 using mental computation. Jackson (2019) on the other hand presented on his YouTube channel the squares of all two-digit numbers using tricks such as squaring the individual numbers separately and adding the result to the product of the two-digit numbers. Other experts in squaring numbers used the grid, middle sum, place value, and math magic to perform the products (Gardner, 2011; Jensen, 2012). Their contributions to the topic inspired the researchers to venture into other solutions in squaring numbers. They made lots of research and came up with the idea to use the sequence as an alternative formula for squaring two-digit numbers.

Design and development of alternative solutions and finding the rule or formula were presented next. In getting the formula, the researchers used the patterns in a sequence to find its rule. A sequence is a function whose domain is the set of positive integers. It also means an ordered list of numbers. In a sequence, each element is called a term, which corresponds to a natural number. Normally, a sequence has a rule or formula for the value of its terms. However, there are some sequences that the rule that cannot be easily determined so it requires a more in-depth examination of the given data. To be able to get the formula or rule, constructivist learning theory is involved. For that circumstance, we must find a recurrence relation. A recurrence relation defines the relationship between successive terms (Nivera & Lapinid, 2015).

The idea of finding a rule in a given sequence pushes the researchers to arrive at the following formulas covering the square of numbers from 10-19, 20-29, and 30-39:

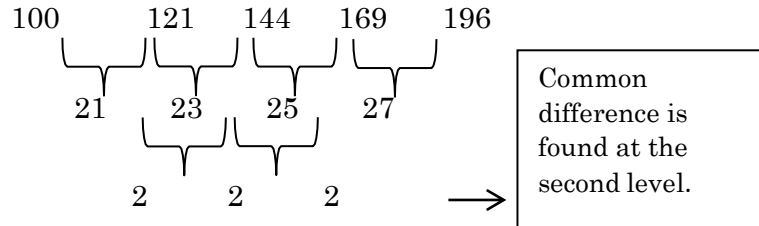
$$\begin{aligned}10^2-19^2 &= n^2 + 18n + 81 \text{ or } (n+9)^2 \\20^2-29^2 &= n^2 + 38n + 361 \text{ or } (n+19)^2 \\30^2-39^2 &= n^2 + 48n + 831 \text{ or } (n+29)^2\end{aligned}$$

First, the researchers analyze the given squares from 10-19.

$$\begin{aligned}10^2 &= 100 \\11^2 &= 121 \\12^2 &= 144 \\13^2 &= 169 \\14^2 &= 196 \\15^2 &= 225 \\16^2 &= 256 \\17^2 &= 289 \\18^2 &= 324 \\19^2 &= 361\end{aligned}$$

Example:

Using the five (5) squares above as a sequence of numbers, {100, 121, 144, 169, 196}

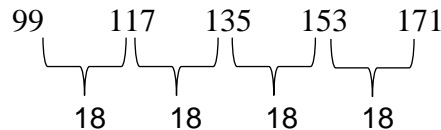


Since the constant difference is at the second level, n^2 is now part of the rule. Now, we multiply the common

n	1	2	3	4	5
a_n	100	121	144	169	196
n^2	1	4	9	16	25
$a_n - n^2$	99	117	135	153	171

difference by $\frac{1}{2}$, so we get, $(\frac{1}{2})(2) = 1$. The rule is now n^2 , then we compare the given result with the given sequence.

From the difference between the given sequence (a_n) and n^2 rule, we can now get a common difference:



Therefore, $18n$ should be added to the rule. We now have $n^2 + 18n$, which we use once again to compare with the given sequence.

n	1	2	3	4	5
a_n	100	121	144	169	196
n^2	1	4	9	16	25
$a_n - n^2$	99	117	135	153	171
$18n$	18	36	54	72	90
$n^2 + 18n$	19	40	63	88	115
$a_n - (n^2 + 18n)$	81	81	81	81	81

We can now get the correct rule by adding 81 to the previous rule, $n^2 + 18n$. Thus, the rule is $n^2 + 18n + 81$.



Following the same process, the researchers can get the rule for the squares of 20-29 ($n^2 + 38n + 81$) and 30-39 ($n^2 + 58n + 361$).

Now, the researchers observed that the derived formula is factorable since the given expression is a *perfect square trinomial*, $n^2 + 18n + 81$. Thus, it can be factored to $(n + 9)^2$. Same with the two formulas, $(n + 19)^2$ and $(n + 29)^2$. Note that n is the distance of the given number from its starting point.

To get the other formulas, you just need to add 10 to the second term. Thus,

Case 1: $10^2 - 19^2 = (n+9)^2$

Case 2: $20^2 - 29^2 = (n+19)^2$

Case 3: $30^2 - 39^2 = (n+29)^2$

Case 4: $40^2 - 49^2 = (n+39)^2$

Case 5: $50^2 - 59^2 = (n+49)^2$

...so on and so fort

The last phase was the experts' evaluation of the rule or formula discovered. It determined the effectiveness of the alternative solution. This was done through the adopted instrument from Subia (2018). Lastly, comments and suggestions from experts were transcribed to improve the research and the created learning resource.

1.3. Objectives of the Study

This study developed a new alternative solution to solving a square of numbers. Specifically, this study aimed to:

1. describe the development of the alternative solution or formula.
2. determine the effectiveness of the alternative solution described by the experts.
3. determine the ways to improve the alternative solution based on the feedback of the respondents; and
4. discuss the implications of the study to the teaching and learning of mathematics.

2. Method

2.1. Research Design

This study used a descriptive-developmental research study that aimed to develop an alternative solution using the patterns in a sequence and to describe and evaluate the effectiveness of the formula introduced by the researcher. Furthermore, it was used as an evaluation study since the evaluation of the experts on the effectiveness of the alternative solution was determined. According to Powell (2006), as cited by Subia (2018), the goal of the evaluation study is to analyze a certain practice or intervention at a specific location. Its main goal is to assess the effectiveness of interventions in the real world, such as new treatment approaches or innovations. Evaluation study, it is argued, is a systematic



procedure that entails gathering data regarding organizations, processes, programs, services, and/or resources. Knowledge and understanding should be enhanced through evaluation studies making decisions and resulting in practical implementations.

Various research locales were identified as the place in conducting the study. These were the schools found in the Schools Division of Cabanatuan City and Nueva Ecija, but in this study, the Basic Education Department of Wesleyan University-Philippines was the primary source of data.

2.2. *Sampling*

This research was evaluated purposively by selecting 20 respondents that meet the following criteria: all have taken the topic, Square of a Number, come from different ages, have professional status, and are willing to lecture and evaluate the discussion made by the researcher for not more than 30 minutes. According to Suen et al., (2014), purposive sampling is the purposeful selection of a participant based on the participant's characteristics-

These experts of evaluators were teachers, head teachers, supervisors, program coordinators, heads, and former heads who have experience in teaching in the field of mathematics. All the respondents were from the Schools Division of Cabanatuan City and Nueva Ecija.

This research's participants were teachers, head teachers, supervisors, program coordinators, heads, and former heads. The criteria for selecting the respondents included their highest educational attainment (Licensed Professional Teacher, with MA Units, Doctoral Units, Master's degree holder, and Doctorate degree holder), years in service, aligned in the field of mathematics, and had to experience in teaching square of numbers. They were to respond to the survey questionnaire and evaluate the researcher's developed new formula for squaring a number through patterns in a sequence. The following is the list of experts that evaluated the alternative formula discovered by the researcher using the patterns in a sequence.

2.3. *Research Instrument*

The utilization of a Survey Questionnaire was the primary tool used in this investigation. Questionnaires are made up of a series of questions that are used to collect a great amount of data from responders rapidly and effectively (Macleod, 2018).

Meanwhile, the employed instruments were composed of three parts: (1) the discussion on the topic, "Solving a Square of a Number through Patterns in a Sequence, (2) the evaluation rating sheet adopted from Subia (2018) wherein 12 items were included to evaluate the effectiveness of the alternative solution introduced by the researcher and (3) comments and suggestions of experts on the crafted intervention of solution.

The tool used in this study is a four-point Likert scale to carefully evaluate the effectiveness of the alternative formula in squaring a number.



Scale	Verbal Interpretation	Remarks
4	Strongly Agree	Very Effective
3	Agree	Effective
2	Disagree	Not Effective
1	Strongly Disagree	Worst

Meanwhile, the experts' comments and suggestions to improve the learning resource were in the form of a response-type questionnaire. The researcher used the transcription way of interpreting respondents' responses.

2.4. Data Gathering Procedure

The following steps were observed in the conduct of the study:

Permission was secured from the Dean of the College of Education, Graduate Studies of Wesleyan University-Philippines regarding the study's conduct. The researchers developed an alternative solution and was able to discover a formula for Squaring a Number using Patterns and Sequences based on the reviews of existing research and discoveries of other experts. But, the researchers uniquely improvised a formula using Patterns in a Sequence that made his discovery different from others.

An evaluation rating sheet for the effectiveness of the alternative solution introduced by the researcher was adopted from the study, "Comprehensible Technique in Solving Consecutive Numbers" (Subia, 2018), to evaluate and draw essential data about the developed formula for Squaring a Number. A complete list of the target respondents was secured, and respondents were selected thru purposive sampling. Then, a recorded video lecture was provided to the respondents to those who were not able to attend the synchronous discussion due to important matters.

Also, permission was asked from the principal of the Basic Education Department of Wesleyan University-Philippines, and individual permission for the selected respondents. The researchers retrieved the survey forms, tallied their answers, and then analyzed the results.

2.5. Data Analysis

The researchers collated, categorized, and arranged the different data sets collected. The data collected were statistically treated to solve the study's unique challenges. This study used the following data analyses:

1. to describe the development of an alternative solution or the formula for squaring numbers, the researchers conducted several analyses of existing formulas or

2. techniques for solving square numbers. They look for patterns to devise new formulas in squaring numbers.
3. to describe the effectiveness of the alternative solution introduced by the researchers in solving a square of a number. The researcher used the rating scale questionnaire to gather the necessary information and weighted mean to analyze the given set of data.
4. to describe the ways of improving the alternative solution, the comments, and suggestions of the respondents for the improvement of the technique introduced by the researcher in solving a square of a number, the researchers used the transcription of texts to evaluate respondents' responses.
5. to discuss the implications of the study in the teaching and learning of mathematics, the researchers identified implications of the study based on the results.

3. Results

3.1. Development of Alternative Solution Using Patterns and Sequence

The idea of finding a rule in a given sequence pushes the researchers to arrive at the following formulas covering the square of numbers from 10-19, 20-29, and 30-39:

$$10^2-19^2 = n^2 + 18n + 81 \text{ or } (n+9)^2$$

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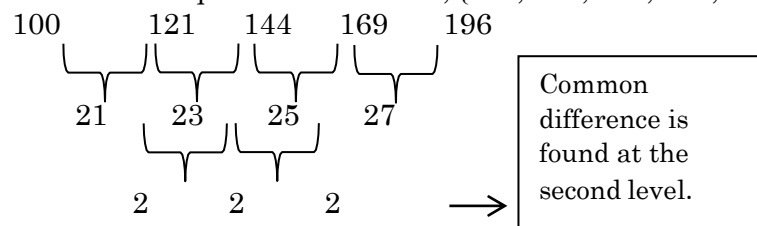
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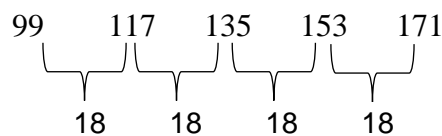
Example:

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Since the constant difference is at the second level, n^2 is now part of the rule. Now, we multiply the common difference by $\frac{1}{2}$, so we get, $(\frac{1}{2})(2) = 1$. The rule is now n^2 , then we compare the given result with the given sequence.

From the difference between the given sequence (a_n) and n^2 rule, we can now get a common difference:



Therefore, $18n$ should be added to the rule. We now have $n^2 + 18n$, which we use once again to compare with the given sequence.

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Following the same process, the researcher can get the rule for the squares of 20-29 ($n^2 + 38n + 81$) and 30-39 ($n^2 + 58n + 361$).

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...so on and so fort

3.2. Effectiveness of the Alternative Solution

Table 1. Effectiveness of the Alternative Solution

	WM	VI
1) The technique is engaging and enjoyable.	3.75	SA/VE
2) The technique is innovative and creative.	3.90	SA/VE
3) The technique is timely and relevant.	3.80	SA/VE
4) The technique applies to any type of learner.	3.40	SA/VE
5) The technique can motivate the learner to study Mathematics.	3.55	SA/VE
6) The technique is easy to use.	3.70	SA/VE
7) The technique is useful to the learner who does not like Mathematics.	3.50	SA/VE
8) The technique is meaningful to the learner.	3.65	SA/VE
9) The technique provides aids in dealing with the square of a number without the use of the calculator.	3.80	SA/VE
10) The technique is direct.	3.70	SA/VE
11) The technique provides opportunities for students to learn	3.75	SA/VE
12) The technique satisfies various learning styles.	3.60	SA/VE
Overall Weighted Mean	3.68	SA/VE

Legend: 1.00-1.74 Strongly Disagree-SD (The technique is worst); 1.75-2.49 Disagree-D (The technique is not effective); 2.50-3.24 Agree-A (The technique is effective); 3.24-4.00 Strongly Agree-SA (The technique is very effective).

Table 1 shows the respondents' rating on the effectiveness of the perspicuous technique introduced by the researchers. It can be obtained from the table that the respondents strongly agreed with the effectiveness of the technique, this means that the technique being introduced was very effective (OWM=3.68). They rated particularly that it is a very effective innovative and creative technique (WM=3.90). Moreover, the respondents rated the technique as timely and relevant (WM=3.80) since it is new to the learner's view, it provides aids in dealing with the square of a number without the use of a calculator (WM=3.80) since the formula used basic calculations, it is engaging and enjoyable (WM=3.75), and it provides an opportunity for students to learn (WM=3.75). The results showed that the items mentioned on the effectiveness of the alternative solution based on the respondents' perceptions were very effective.

Based on the respondents' rating of the effectiveness of the alternative solution in squaring of a number, the technique is an excellent alternative formula for squaring



numbers. In the study conducted by Ossiannilsson (2018), he found that student engagement, retention, and academic results can all benefit from the use of innovative learning methodologies and learning spaces. It is critical that learning environments can adapt to future demands and continue to provide world-class teaching and learning opportunities through the design and implementation of cost-effective, adaptable, and future-based spaces. Innovation in learning mathematics such as providing alternative solutions or formulas can help students engage and develop their interest in Mathematics.

3.3. Feedback from the Respondents

The followings are the comments and suggestions of the respondents regarding the technique introduced by the researchers:

R1: *“The technique given can be enjoyed and used by students, especially those who usually join competition”*

R2: *“I could say that this technique in finding the square of a number can be taught and be applied for higher grades only. Pupils might get confused about the step-by-step solution to find the square of a number rather than using the traditional way of solving of a number. Learners must have first knowledge of the concept of getting the square of a binomial before applying this technique, But, I think for some learners to love math will appreciate this new technique.”*

R3: *“The technique is easy to use, direct, and simple and can be used as a shortcut in finding the square of a number.”*

R4: *“For the fast learners, I think this method is very much effective, for sure they will enjoy learning solving a square of a number using this method, but for some learners who don’t care about memorizing formula and square of numbers ending in 9, I don’t think they will appreciate it. But the learners’ performances will depend already in the motivation of the teacher. For mathematics teachers like me, will surely appreciate your excellence in discovering this technique in solving a square of a number through.”*

R5: *“I have been teaching Mathematics for almost 10 years. Upon hearing the author’s discussions about the different way of solving a square of a number, it amazed me because it’s precise and much easier. I am sure that I will use this formula from now on as it may help the learners who are having a hard time solving a square of a number. Big thanks and advance congratulations to the author! More power!”*

R6: *“The process is very easy and fun. It just needs to be explained carefully so the students can really understand the process.”*

R7: *“This new pedagogical strategy is unique and simple.”*

R8: *“The illustrations and discussions on how to arrive to the pattern is very clear, easily understand and the learners will enjoy learning math the easy way.”*



R9: *“The technique is innovative for learners in the junior and senior high school. It is applicable for above-average learners since it involves series and sequences. It will be helpful for STEM students since subjects under STEM are higher mathematics and sciences. The technique is not applicable to all types of learners.”*

R10: *“Thank you for giving me the opportunity to be respondents of your research. As I listened to the discussion, I felt the writer’s excellence and dedication to his field of specialization. The topic will help to address learner’s diversity if it can be introduced by math teachers to learners. It is a way for them to see the beauty of math meaningfully, i.e., there are many ways to explain and demonstrate math concept.”*

R11: *“It’s a very good idea for solving a square of a number. It’s easy and precise. Students will surely like and enjoy using this method. The author is excellent with this accomplishment. This may help learners learn mathematics the easier way. Highly commendable.. congratulations!!”*

R12: *“The researchers’ technique in showing another way on finding the square of a number (esp. finding the perfect square of bigger numbers) is easy to understand & well discussed.”*

R13: *“The researchers’ technique in showing another way on finding the square of a number (esp. finding the perfect square of bigger numbers) is easy to understand & well discussed.”*

R14: *“Start dreaming or making your own workbook for the Mathematics students in the Senior High School and Tertiary level students. Keep up the good works and God bless you more and always!”*

R15: *“The technique is very creative and innovative. I tried to use the formula in many cases and it was proven true. Therefore, I may say that this technique is very efficient also.”*

R16: *“This inspires me to discover also an alternative solution in some of my mathematics classes.”*

R17: *“The technique is a good way to introduce to the learners having difficulty squaring of numbers especially if larger numbers involved. The technique is direct and simple.”*

R18: *“You can still improve this. Just don’t forget to cite sources every time to emphasize the difference of your discovery to others.”*

The perspicuous technique was proven to be innovative and creative, timely and relevant, aids learners in dealing with the square of a number without using a calculator, and provides opportunities to learners in squaring numbers based on the transcriptions of the respondents’ responses. In comparison to the traditional or algorithm way of multiplying two same numbers, it is a very effective and efficient strategy since the formula can cover all of the numbers. Similarly, because the technique incorporates basic multiplication and addition, it can be used by learners who will participate in Math competitions.

Respondents 2, 4, and 9 had common perceptions that the technique was very effective. However, this technique is much more enjoyable for higher students or advanced learners.



Other responses were amazed and commented on how the formula was developed using the patterns in sequence. Some of them were inspired also to venture into developing alternative solutions. Meanwhile, respondent 7 stated that the new pedagogical strategy is unique and simple. On the other hand, respondent 6 suggested that the technique should explain to the learners carefully so that they can use it properly. Lastly, appreciation and congratulatory messages were stated because of the researchers' new discovery.

3.4. Implications of the Study to the Teaching and Learning Mathematics

The alternative solution introduced by the researchers, which was found very effective, innovative, and creative, implies that the learning of the students depends mostly on the teaching methodology set by the teacher. Therefore, as teachers, we need to be creative and innovative to give quality education to learners. This new intervention provides the learners with an avenue to learn that is different, convenient, and easy from the traditional methods.

The result of the study implied that in the teaching and learning process, we must provide alternative ways of learning so that the students will be engaged in mathematics learning. It is clearly implied based on the findings and respondents' perceptions that students tend to learn if they have other ways of solving a sample problem. Thus, teachers will look to some patterns to reverse the complex method in an easy and convenient way.

Lastly, teaching mathematics does not start and end with what is being written in the textbook. Teaching mathematics requires a lot of strategies that the teacher needs to use. For teachers to make it easy to teach math, they must provide a variety of strategies as well as alternative methods. For students to be able to learn, complex mathematics can be perspicuous in a way that they are enjoying the process of learning. Thus, for them to enjoy, innovations such as discovering new methods help them fighting anxiety towards mathematics.

In the study conducted by Munna A. S. & Kalam M. A. (2021), what they found most fascinating about a learning session and what results they hoped to attain from the teaching-learning session based on their prior understanding of student learning. Varied students provided different responses to these questions, indicating that different students utilized different strategies, and diverse learning styles to participate actively in the teaching-learning session and achieve meaningful learning outcomes. They also deduced from the students' responses that learning outcomes are also influenced by learner types. There are several different sorts of learners in a classroom context. The group is of them aural learners who are more confident in their ability to receive and process auditory stimuli. The instructions that students receive through classroom lectures, discussion sessions, and group sessions assist this category of learners the most.

In this regard, (Coffield, 2013) claims that the quality of teaching and learning is frequently harmed owing to a lack of continuous knowledge up-gradation on the subject or issue. We agreed with this perspective since we have seen first-hand how innovation and continual learning are the most important factors in educational effectiveness.

4. Discussion



The perspicuous technique (PT) was investigated and examined as an alternate and innovative method for solving squares of numbers in Mathematics. Based on the findings of this study, the following conclusions were drawn:

The derivation of the formula in squaring of numbers was from the analysis of finding the patterns and sequence.

The result on the effectiveness of the alternative solution or formula was rated by the respondents as very effective, the technique is innovative and creative, timely and relevant, aids learners in dealing with solving squares of numbers, engaging and enjoyable, and provides opportunities for students to learn. Therefore, the perspicuous technique can be used as an innovative tool to solve squares of numbers since it covered the numbers in Mathematics and increases students' engagement to learn mathematics.

The respondents' comments and suggestions on the effectiveness of the alternative solution were very effective, easy, innovative and creative, timely, and relevant, aids learners in dealing with the square of a number without using a calculator, and provides opportunities to learners in squaring of numbers. Some respondents had the same perception regarding the students of enjoyment using the formula, especially the advanced or higher-grade learners. Some of them were inspired, amazed, and commended the discovery of the formula using the patterns in a sequence. They also stated that the new pedagogical strategy is unique and simple, but needs to be explained carefully to the learners. Thus, the alternative solution introduced by the researcher can help learners in dealing with squaring of numbers. Also, proven effective and efficient.

The study had implications for the teaching and learning process that teaching methodology and strategy affect students' process of learning. Therefore, teachers must provide or venture on various strategies especially in providing alternative solutions that cannot be found in the textbooks. Also, teaching and learning of mathematics will become perspicuous if teachers turn complex computations to simple one.

5. Recommendations

In light of the conclusions, the following are hereby recommended:
The number of experts in evaluating the effectiveness of the alternative solution or technique should be increased.

It is recommended that the authors should convey the perspicuous technique (PT) to his colleagues, friends, peers, other Mathematics instructors or lecturers, and engineers so that the technique can be widely disseminated and explored. They can share this with their pupils or students and take the lead in developing strategies that will serve as alternate solutions in dealing with complex Mathematical problems, both to meet the requirements of individuals who are not mathematically inclined and to those who enjoy Mathematics.

It is suggested to the people aligned in mathematics especially teachers who are teaching mathematics to venture into finding some ways of alternative to traditional practices to help the students engage, enjoy, and love to learn Mathematics.



Teachers are also suggested to innovate their pedagogical strategies so that the learners will engage and enjoy Mathematics.

It is suggested to future researchers to invest and conducts a study on innovative research that could benefit the community.

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