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International Journal of Curriculum and Instruction 16(2) (2024) 434–440

IJCI
International Journal of Curriculum and Instruction

Using crossing method for teaching, learning and solving systems of linear equations for two unknowns that yield no solution in Tanzanian secondary schools

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Abstract

This paper presents an alternative approach of teaching, learning and solving systems of two linear equations for two unknowns that yield no solutions. We conducted a desk-based research on methods that have been used by in-service mathematics teachers for teaching systems of two linear equations in ordinary secondary schools. It was found that five common methods (substitution, elimination, graphical, inverse matrix, Cramer's rule) have been used for teaching a system of two linear equations for two unknowns that yield no solution and all methods yield the same answer regardless of having different ways of approaching the system. We realized that a crossing method (alternative approach) is not found in the literature and yet not used by teachers for teaching students a system of two linear equations that yield no solution. But this crossing method yields similar answers with that resulted when using the five common methods. We present this alternative method in this paper by comparing with the answers obtained using five methods while focusing on two systems of two linear equations that yield no solution. This new approach has implications in teaching, learning and solving systems of two linear equations that yield no solutions in ordinary secondary schools, including mathematics teachers and educators can use this method for teaching students in solving systems of two linear equations for two unknowns that yield no solutions.

Keywords: Mathematics teachers; Crossing method; Systems of two linear equations; No solution

1. Introduction

A system of linear equations is one of the fundamental concepts in algebra which involves finding the values of the variables which tend to satisfy the multiple equations simultaneously (Deogratias, 2022; Tanzania Institute of Education, 2021). When dealing with systems of linear equations, it is important to understand the relationship which exists between the equations as these systems may have unique solutions, infinitely many solutions and there occurs a

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situation in which these systems have no solution depending on the slopes and the y-intercepts of the equations.

A system that yields no solution happens when you are dealing with equations that represent the parallel lines in a coordinate plane. These lines actually never cross paths and as a result there is no chance of finding a shared point of intersection. These lines are like two friends walking side by side, but they are on paths that will never intersect. Now, the key reason for this lack of solution lies in the parallel lines having the same slope but different y-intercepts. The lines bring no solution no matter how you fiddle the equations or tinker with the variable values; there is no point on the coordinate plane where these lines will intersect. It is an interesting concept and quite a clear indication that a solution is unattainable in this case.

The crossing method often referred to as the cross-multiplication method, is a clever alternative approach in the realm of solving systems of linear equations (Deogratias, 2022). It offers an intuitive way to find the values of the variable that satisfy both equations simultaneously. It stands out as one of the simplest yet remarkable methods that give precision and accuracy to the value of the variables. In essence, the crossing method offers an excellent entry point for clear understanding and working with linear equations (Deogratias, 2022). This method produces identical results to those achieved when solving linear equations using elimination, graphical, substitution, Cramer's rule and inverse matrix method as elaborated in this paper.

In the Tanzanian mathematics curriculum for secondary schools, the instruction of systems involving two unknown linear equations relies on a set of methods including elimination, substitution, Cramer's rule and the use of inverse matrix method (Tanzania Institute of Education, 2005, 2010, 2021). As per Deogratias (2022), the instructional approach in Tanzanian secondary schools is structured such that students are introduced to elimination, substitution, and graphical methods during their Form I mathematics classes. This foundational understanding lays the groundwork for advanced concepts. It is in Form IV during their tenure in ordinary secondary schools that students delve into the complexities of inverse matrices and Cramer's rule which serve as a higher-level component of the curriculum (Tanzania Institute of Education, 2005; 2009a; 2009b, 2010, 2021). The interesting aspect is that, despite their different approaches, all these methods converge to the same solution (Deogratias, 2022).

2. Materials and methods

In the course of this study, we embarked on a desk-based research endeavor to examine the existing body of literature concerning the techniques employed for solving systems of two unknown linear equations. Our exploration extended beyond the confines of Tanzanian secondary schools and reached into the realms of higher education encompassing universities, diploma colleges and vocational training institutions.

Desk-based research revealed that Tanzanian secondary schools primarily employ five well established methods: elimination, substitution, graphical representation, Cramer's rule and inverse matrix method to teach students the concepts of solving systems of linear equations with two unknowns (Deogratias, 2022; Tanzania Institute of Education, 2005; 2009a; 2009b, 2010, 2021). Recognizing this prevalent pedagogical landscape, this paper introduces an innovative approach "the crossing method" designed to enrich the teaching of systems of two linear equations in secondary education.

This paper will provide a practical demonstration of this alternative approach through the analysis of a specific system of two linear equations. Furthermore, we will present two additional examples involving systems of two linear equations, offering comparative illustrations of the answers obtained through using the crossing method and the five conventional techniques, which include, elimination, substitution, graphical representation, Cramer's rule and inverse matrix method. These illustrations aim to shed light on the efficacy and applicability of this new teaching approach.

3. Describing a Crossing method for teaching systems of two linear equations for two unknowns

Consider the following system of linear equations in two variables:

$$a_1x + b_1y + c_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

$$a_2x + b_2y + c_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Step (1) Equation (1) is multiplied by b_2 and Equation (2) is multiplied by b_1

$$b_2a_1x + b_2b_1y + b_2c_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$b_1a_2x + b_1b_2y + b_1c_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Step (2) Subtracting equation (4) from equation (3)

$$(b_2a_1 - b_1a_2)x + (b_2b_1 - b_1b_2)y + (b_2c_1 - b_1c_2) = 0$$

$$\Rightarrow (b_2a_1 - b_1a_2)x = (b_2c_1 - b_1c_2)$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{b_2a_1 - b_1a_2} \text{ given } b_2a_1 - b_1a_2 \neq 0$$

Step (3) The value of x obtained as such is substituted either in equation (1) or equation (2).

Hence the value of y obtained is $y = \frac{c_1a_2 - c_2a_1}{b_2a_1 - b_1a_2}$ given $b_2a_1 - b_1a_2 \neq 0$

4. Illustrations of the Crossing method for teaching systems of two linear equations

To put this concept into practice, let us demonstrate it by working through the solution of the following system of linear equations.

$$x + 2y = 4$$
$$3x + 6y = 18$$

Given that $a_1 = 1, b_1 = 2, c_1 = 4$ and $a_2 = 3, b_2 = 6, c_2 = 18.$

We know that $x = \frac{b_1c_2 - b_2c_1}{b_2a_1 - b_1a_2} = \frac{36-24}{6-6} = \frac{30}{0}$ and $y = \frac{c_1a_2 - c_2a_1}{b_2a_1 - b_1a_2} = \frac{12-18}{6-6} = \frac{-6}{0}$.

Since the result obtained is undefined, this means that the system has no solution.

5. Solving systems of two linear equations using crossing method by comparing the answers with those obtained through substitution, elimination, graphical and inverse matrix methods, and Cramer's rule

In this part we are going to use five common methods and the crossing method to solve a system of two linear equations:

$$\begin{cases} 2x + 3y = 5 \\ 2x + 3y = 7 \end{cases}$$

5.1. Solving the above system of linear equations by using crossing method

Given that $a_1 = 2, b_1 = 3, c_1 = 5$ and $a_2 = 2, b_2 = 3, c_2 = 7$

From $x = \frac{b_1c_2 - b_2c_1}{b_2a_1 - b_1a_2} = \frac{21-15}{6-6} = \frac{6}{0}$ and $y = \frac{c_1a_2 - c_2a_1}{b_2a_1 - b_1a_2} = \frac{10-14}{6-6} = \frac{-4}{0}$.

Since the result obtained is undefined, this means that the system has no solution.

We will now compare this result with what we would obtain using elimination, substitution, graphical representation, Cramer's rule and the inverse matrix method.

5.2. Solving the system of linear equations by using the method of elimination

Considering our system of linear equations

$$\begin{cases} 2x + 3y = 5 \\ 2x + 3y = 7 \end{cases}$$

As the coefficients of x and y are identical in both equations, we can proceed by subtracting Equation(1) and Equation(2), and when we do this we will notice that, the left hand side of both equations vanishes, resulting in $0 = -2$, which is not true. This means that the system of linear equations has no solution.

5.3. Solving the system of linear equations by using the method of substitution

We will demonstrate the second method for solving a system of linear equations using the same example we employed in the elimination method.

Let us go back to the linear equations:

$$\begin{cases} 2x + 3y = 5 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5) \\ 2x + 3y = 7 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6) \end{cases}$$

Upon organizing equation (5), we can observe that, $x = \frac{5-3y}{2} \dots\dots\dots\dots\dots\dots (7)$

Now, if we substitute (7) into (6), we get

$$2\left(\frac{5-3y}{2}\right) + 3y = 7$$

$$\Rightarrow 5 - 3y + 3y = 7$$

$$\Rightarrow 5 = 7$$

Which is not true, and this implies that the system of linear equations has no solution.

5.4. Solving a system of linear equations by using graphical method

In graphical terms, when the coefficients of the variables in our system of linear equations are equal, it signifies that the lines they represent have the same slope. When two equations share this common slope but diverge on the y-axis intercept, they become parallel lines on the Cartesian plane. With no intersection point to be found, it is evident that the system has no solution.

5.5. Solving the system of linear equations by using inverse matrix method

Using the same system of linear equations

$$\begin{cases} 2x + 3y = 5 \\ 2x + 3y = 7 \end{cases}$$

We can write the system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $C = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$. Then $AX = C$.

By introducing and multiplying by A^{-1} both sides of $AX = C$, we have;

$A^{-1}AX = A^{-1}C \Rightarrow X = A^{-1}C$. This gives the solution of a system of linear equations.

From $A = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$, $\det(A) = 0$.

Since the determinant of a matrix $A = 0$, the matrix has no inverse and therefore, the matrix has no solution.

5.6. Using the same system of linear equations to solve it by using Cramer's rule

Consider the system

$$\begin{aligned} 2x + 3y &= 5 \\ 2x + 3y &= 7 \end{aligned}$$

We can write the system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Converting the system into augmented matrix form, it can be written as:

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 2 & 3 & 7 \end{array}\right]$$

Let $D = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$. Next, we proceed to calculate the determinant of the square coefficient matrix, which is located on the left-hand side of the augmented matrix. Determinant, $|D| = 0$.

As the determinant of the matrix equals zero, it follows that the matrix equation has no solution.

Generally, this approach works for other systems of linear equations of two unknowns that yield no solutions.

6. Conclusion

In this paper, we have explored several methods commonly employed by mathematics teachers to facilitate students on solving systems of linear equations with two unknowns that yield no solutions. Our findings have revealed that these methods (elimination, substitution, graphical, Cramer's rule and inverse method) and the innovative "crossing method" yield similar results. Based on this result, we propose the adoption of the crossing method as an alternative approach for teaching students the intricacies of solving systems of two linear equations, not only within Tanzanian secondary schools, but also in educational contexts beyond our nation's borders. This recommendation aims to enrich the repertoire of teaching techniques for this subject, complementing the existing five methods outlined in Tanzanian mathematics textbooks and syllabi: substitution, elimination, graphical, Cramer's rule and inverse matrix method.

References

Deogratias, E. (2022). Using a crossing method as an alternative approach for teaching systems of linear equations in secondary schools. *International Online Journal of Education and Teaching,9(2)*, 1022-1031.

Tanzania Institute of Education. (2021). *Basic mathematics form four*. Dar es salaam, TZ: Tanzania Institute of Education.

Tanzania Institute of Education. (2021). *Basic mathematics form one.* Dar es salaam, TZ: Tanzania Institute of Education.

Tanzania Institute of Education. (2010). *Basic Mathematics syllabus for secondary schools.* Dar es salaam, TZ: Tanzania Institute of Education.

Tanzania Institute of Education. (2009a). *Secondary basic mathematics book one.* Dar es salaam, TZ: Educational books publishers Ltd.

Tanzania Institute of Education. (2009b). *Secondary basic mathematics book four.* Dar es salaam, TZ: Educational books publishers Ltd.

Tanzania Institute of Education. (2005). *Basic Mathematics syllabus for secondary schools.* Dar es salaam, TZ: Tanzania Institute of Education.
