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The importance of using real objects for teaching and learning a mathematical concept with pre-service teachers of mathematics

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Abstract

This paper addresses the ways that real objects are important for conceptual development and understanding of pi. While using real objects in teaching and learning pi with nine university pre-service teachers (PSTs) of Mathematics, the teaching and learning processes were framed under the lens of social constructivist perspective (Vygotsky, 1978). Individual PSTs constructed mathematics and elaborated their understanding of mathematics through social interactions with more knowledgeable others. It was found that PSTs: learned mathematics by doing, interpreted a concept in multiple ways, learned mathematics relationally, learned how to develop the meaning of pi and its value, and real objects helped PSTs eliminate systematic errors. The findings have implications in teaching and learning mathematics, including PSTs can learn how to use real objects in teaching and learning mathematics in their own classrooms.

Keywords: Real objects; Teaching and learning mathematics.

1. Introduction

1.1. *Context*

This paper starts by introducing the challenges of teaching and learning mathematics in the Tanzanian university mathematics classes. Then, the paper presents the theoretical framework used in this study. After that, the paper addresses the ways that teaching and learning pi have been done in the literature to identify the gap. Also, the paper describes the methods used in data collection and analysis. Then, the paper

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presents the results on the importance of using real objects for conceptual development and understanding of pi and other mathematical concepts. The paper ends by providing concluding thoughts on using real objects for teaching and learning mathematics.

1.2. The challenges of using real objects in Tanzanian university mathematics classes

There is a challenge for Tanzanian university mathematics pre-service teachers on teaching a mathematical concept using real objects. This challenge is due to the fact that these teachers are taught in their university classrooms without putting emphasis on how a real object can be used for conceptual development and understanding of a mathematical concept (Deogratias, 2019; 2020). One of the reasons is that university instructors believe that these teachers are competent enough to learn and understand mathematics without using real objects. Also, instructors believe that using real objects for teaching and learning mathematics is time demanding in the design and implementation of the instructional activities. As a result, mathematics is thought by these teachers as an abstract. This study presents the importance of engaging PSTs with mathematics by using real objects for conceptual development and understanding of a concept. Using real objects in mathematics classrooms, PSTs gained both knowledge and skills on how to use real objects for teaching and learning mathematics in their own classrooms after completion of their university degree programs.

1.3. *This study is framed under the lens of social constructivist perspective*

In this study, a socio-cultural approach of the teaching and learning of mathematics is considered by assuming that learners construct their mathematical knowledge through social interactions in a cultural context (Vygotsky, 1978, Taş, 2021). Through interactions, individual and group of learners elaborate their mathematical ideas with knowledgeable others in the zone of proximal development. Also, individual learners get an opportunity to reflect on their learning during teaching and learning process and at the end of the process. Another assumption is that material objects play an important role in knowledge acquisition and understanding of mathematics. For this reason, the importance of using real objects for conceptual development and understanding of pi is presented in this study.

1.4. *Teaching and learning pi*

Pi has been taught to the students in schools and colleges in different ways. For instances, Mason and Roth (2007) used a tube container, tennis balls, a cloth tape measure, a meter-stick and a meter string as teaching and learning resources. They used Archimedes' approach to determine the circumference of the inscribed and circumscribed polygons within the circles. They also used three different teaching and learning strategies: a series of design experiments (Cobb et al, 2003; Lobato, 2003) to engage students with the formula for the circumference. That is, ($C = 2\pi r$) represents the circumference of a circle, and($A = \pi r^2$)represents area of a circle, small group for collaboration (Johnson & Johnson, 2004), and demonstration on how to measure the diameter of a tennis ball, and circumference of the circumscribed and inscribed polygons within the circles. The authors found that the circumference of the circumscribed and inscribed polygons within the circles gave the approximation value of pi, which is approximated to 3.14.

Buhl (2001) used six (6) coins, calculators and spreadsheets as teaching and learning resources. The author used Archimedes' approach to approximate the value of pi by computing the radius of a coin, number of coins, and radius of inner circle. Buhl also used three teaching strategies: the relationship of coins and the radius of an inscribed circle to find the value of pi as an irrational number, posing the following questions to the students "Suppose that six pennies are arranged in a circle, thus creating a circle in the center in a manner such that they "kiss" one another. Will a seventh penny placed in the center of the circle formed by the coins be tangent to all six coins?" and using small group discussions. The author found that the ratio of the number of coins to the radius of inner circle gives the value of pi, which is approximated to 3.14.

Tent (2001) used a tape measure, and circles including oatmeal, salt and baking powder packages as teaching and learning resources for teaching students in secondary schools about circles and the number π. The author used Archimedes' approach to approximate the value of pi. Tent also used two teaching and learning strategies: a tour with students to measure the circumference and diameter of circles, and small group for collaborations when measuring the circumference and diameter of circles. Tent found that using Archimedes' approach gives a better approximation of the value of pi than using a ratio of the two measurements (circumference and diameter) of a circle.

Thomas, Bell and Xiao (2014) worked together on the concept of pi to find the value of pi from the ratio of the circumference to diameter of a circle using drawings of the regular polygons and measurement tools as teaching and learning resources. Then, they used Archimedes' approach to find the number of sides of a polygon, angle at center of a circumscribed and inscribed circle and angle at perimeter of a circumscribed circle. After that, they used two teaching and learning strategies: inscribing and circumscribing the regular polygons in a circle to find the value of pi. Finally, Bell and Xiao (2014) implemented similar activities to the students in their own classrooms. The authors found that the angle at the center of both circumscribed and inscribed circle and angle at the perimeter of a circumscribed circle give the value of pi, which is approximated to 3.14.

Papadopoulos (2013) used classroom experiments to teach students in grade 6 in primary school about pi and its value. Students used computers to inscribe and circumscribe regular polygons inside and around a circle. The author used Archimedes' approach by asking students to find the ratio of area of a square with side-length equal to the radius of the circle. The author found that the ratio of area of a square with side-length equal to the radius of the circle gives the value of pi, which is approximated to 3.14.

Burns (2014) taught pi to the students in middle school using measurement tools and circles. He used two teaching and learning strategies: demonstrations and class discussions in measuring the circumference and diameter of a circle. The author asked students to measure the circumference and diameter of a circle. After that, students were asked to find the ratio of the two measurements (circumference and diameter) of a circle. They found that the ratio of the two measurements of a circle gives the value of pi, which is approximated to 3.14.

Archer and Ng (2016) investigated pi by using scientific method to engage students in grade 6/7 with mathematical modeling. They focused on investigating the relationship between the circumference and the diameter of a circular object. They found that scientific method helped students to engage with mathematical modeling.

Apart from teaching and learning pi, there is a normal routine of celebrating pi every year among teachers, teacher educators, scholars and lovers of pi in the world. This is usually known as "Pi Day March 14 Celebrations". For instance, Brandwein (2019) has provided resources for teaching and learning pi, which she calls "Pi day lesson plans for your classroom" (retrieved from https://www.jewishedproject.org/news/pi-day-lesson-plans-your-classroom). Also, this year 2021, pi has been celebrated by various teachers through working on various activities with children, including providing plenty of circular objects such as pie tins, soup cans, CDs, and coffee cans (retrieved from https://www.weareteachers.com/pi-day-activities/). Children could measure the circumferences and diameters of the objects.

From the reviewed literature, it is found that many researchers have conducted studies on teaching and learning of pi. Moreover, many websites have highlighted teaching and learning resources for teaching and learning pi. It is found that real objects (artificial objects and local resources) have been used to engage students in learning pi. However, no studies that have been conducted to Tanzanian university mathematics pre-service teachers on the importance of using real objects for conceptual development and understanding of pi.

2. Method

This qualitative case study explored the importance of using real objects for teaching and learning a mathematical concept. This study focused on using real objects for conceptual development and understanding of pi as an example of a mathematical concept.

This research study involved four-day research meetings. Day 1 focused on developing the meaning of pi using real objects. Day 2 focused on developing the value of pi using real objects. Day 3 focused on how pi is related to real things available in local environment. Day 4 focused on using real objects to help students in schools understand the concept of pi. In each day long meeting, individual PSTs constructed mathematical ideas and elaborated their understanding through social interactions with more knowledgeable others in small group and class discussions. The interactions are important to help individual or group of learners to understand a concept in the zone of proximal development (Vygotsky, 1978).

In each daylong meeting, there were three sessions. Session 1 was from 9:00—11:00. Session 2 was from 11:30—1:30. The last session was from 2:30—4:00. The three sessions were spaced for breakfast and lunch.

The data was collected using group learning notes to collect mathematical ideas that were discussed in small groups and then written on a manila sheet for small group presentations. Reflective journals to collect individual reflections about their ongoing learning of mathematics. Audio recordings were used to collect mathematical ideas that PSTs discussed in small groups before writing the ideas on the manila sheet. Video recordings were used to collect mathematical ideas during class discussions.

During data analysis, the audio and video recorded data was transcribed. After that, all collected data was analyzed using thematic analysis (Clarke & Braun, 2006) focused on the importance of using real objects for conceptual development and understanding of pi in the four-day research meetings.

3. Results

The results on the importance of using real objects for conceptual development and understanding of pi are presented based on: PSTs learned mathematics by doing, PSTs related mathematics with real objects available in local environment, PSTs interpreted pi in multiple ways, PSTs learned pi relationally, PSTs learned how to develop the meaning of pi and its value, and real objects helped PSTs eliminate systematic errors. The sub-sections below describe these results in detail.

3.1. *Learning mathematics by doing*

Ben-Hur (2006) argues that learners should learn mathematics by doing—practices. Practices involves all mathematical activities that learners perform in a learning environment, including demonstrations and presentations. Practices are potential to help learners develop knowledge and skills, foster active learning and achieve learning mathematics (Ben-Hur, 2006). Also, through giving an opportunity for learners to practice mathematics, teachers can assess learners' understanding of mathematics in the learning environment. This is because practice is usually visual oriented such as learners' demonstrations of a mathematical concept (Deogratias, 2020).

Before participating in the research meetings, PSTs were learning mathematics in a university mathematics class by relying heavily on lecturing. This involved listening to the instructor while taking notes. They practiced mathematics after lecture sessions through homework assignments. However, after participating in the research meetings, PSTs learned mathematics by doing. Real objects were important to foster this learning throughout the research meetings. For instance, PSTs demonstrated how to measure the circumference of a circular object using a string. For instance, Group 1 presenter describes:

> This is our circular object [a stem of pawpaw], okay! So, when we want to measure the circumference of this circular object, we can use this string. We will identify the starting point using a pencil. For instance, this is my starting point, okay. Then, I will take this string. I will start from the point identified from here, rounding! My string started from the point identified which is this one, okay. I started here to this point around the string to turn back to the starting point. So, the distance of this string from the starting point around a stem of pawpaw to turn back to the starting point will make one complete circle which is the circumference of a stem of pawpaw. (Transcribed data).

Apart from using real objects to learn how to find the circumference of a circular object, the objects were potential during micro-teaching. PSTs used real objects to identify circular objects from non-circular. Identification of circular objects are important in order to measure the circumference and diameter of a circular object using measurement tools such as a string, a ruler and a set of Vernier calipers. This identification of objects can be evidenced from Group 1 presenter in the quote below.

> G1P: Today's session, we are going to learn about the concept of pi [while writing on a board]. About learning the concept of pi, we start identifying circular objects. Now, in a group of three students, identify circular from non-circular objects. Give at least two circular objects. Discuss and write your ideas on a manila sheet. Two minutes group work discussions.
>
> Group 1: Examples of circular objects are stems of cassava and maize.
>
> Group 2: Stems of sugar cane and sunflower. (Transcribed data)

Throughout the four-day research meetings, PSTs learned mathematics by doing. They practiced mathematics during small group presentations, designing a lesson plan for teaching pi by using real objects and performing micro-teaching for teaching pi as a concept. Because of that, pre-services realized the opportunities of giving students to learn pi by doing. This kind of learning can be achieved through using circular objects for conceptual development and understanding. The participant presents:

> Students should be taught by doing on how to find the value of pi starting from primary school. They should be taught how to measure lengths of the circumference and diameter of a circular object. They have to be taught by using different objects which make use of pi. (Reflection)

From the above quote, it seems that this pre-service teacher was not taught in this way in primary school, as well as in secondary schools. Because of that, this teacher sees the opportunity of using circular materials for teaching the concept of pi in primary, secondary, and tertial education levels. As such, using real objects is critical in teaching and learning mathematics in a learning environment.

3.2. *Relating mathematics with real objects available in local environment*

Ben-Hur (2006) suggests that learners need to relate the concepts with what they see in their life situation. This is important to avoid rote among learners. Also, learners can easily connect mathematics with other fields such as sciences. In doing so, real objects were potential in the research meetings to help PSTs easily connect mathematics with what they see in their local environment. PSTs realized that pi is found everywhere in a circular object. For instance, G1 presents:

G1: Mysterious of the number π is that it appears everywhere in circular objects in the world. It is an irrational number $(3.14\ldots)$. (Group learning notes)

Apart from realizing that pi appears in a circular object, PSTs realized the potential of using real objects in teaching and learning mathematics. For instance, PSTs noticed that pi appears in many circular objects that can be improvised in the classroom for teaching and learning pi. For instance, G2 suggests:

G2: We can use various circular objects not only one circular object. Circular objects are many including tapes, stems of pawpaw, and stems of sugarcane. (Group learning notes).

Before participating in the four-day research meetings, PSTs were not aware on how to develop the concept of pi using real objects. They thought that pi and its value is expressed on the relationship of circumference of a circle to its diameter only. Also, the value of pi is always given, which is equivalent to 3.14. However, after participating in the research meetings, PSTs realized that pi and its value can be obtained in circular objects available in their daily environment. PSTs measured the circumference of a circular object using a string and a ruler. They also measured the diameter of the object using a set of Vernier calipers. The measurements of the circumference and diameter of the object give the value of pi, because a ratio of the two measures (circumference and diameter) gives the value of pi as an irrational number. In this case, circular objects were used as thinkable resources to develop the concept of pi and its value in the research meetings. Hence, real objects changed PSTs' notions of seeing that mathematics is an abstract. They realized that mathematics is connected to real objects available in their own environment.

3.3. Multiple interpretations

Before conducting four-day research meetings, PSTs thought that there is only one definition of pi. They thought that pi is a ratio of circumference of a circle to its diameter. However, after participating in the research meetings, PSTs realized multiple ways of defining pi. This was achieved through using real objects in identifying circular objects from non-circular; measuring the circumference and diameter of a circular objects using measurement tools such as a string, a set of Vernier caliper and a rule; and finding the ratio of the two measures (circumference and diameter) of the object.

G1: Pi is the ratio of an angle turned to the length of an arc. (Transcribed data)

G3: Pi refers as a ratio between the circumference of a circle and its diameter. i.e. $\text{pi} = \frac{\text{Circumference of a circle}}{\text{Diameter of a circle}}$. (Group learning notes)

G2: Pi is the number of diameters of a certain circle to complete one circumference. (Group learning notes)

From the above quote, G1 defined pi by relating an angle measure in a circle with arc length. This relationship is due to the fact that the length of an arc subtended on a circle is proportional to the measure of the central angle. G2 defined pi by relating a circumference and diameter of a circle. This relationship is important because the ratio of a circumference of a circle/circular figure/circular object to its diameter gives the value, which is equivalent to 3.14. G3 defined pi by relating the counting process of the number of diameters required to complete the circumference of a circle. This relationship is important because the number of diameters around the circumference of a circle gives that value of pi. This value is an irrational number (3.14 ……).

3.4. Relational learning and understanding of pi

Skemp (1976/2006) suggests that learners need to understand mathematics relationally rather than rote. Skemp defines relational understanding as knowing how and why a concept makes sense. This kind of learning leads to learners' remembering information. It is easy to retrieve information because of high retention. He also defines rote understanding as knowing a concept without realizing why a concept makes sense. This kind of understanding leads to learners' memorizing procedures and facts. It is also easy to forget the learned concept due to poor retention.

In the research meetings, PSTs learning mathematics relationally. This kind of learning was easily fostered by using real objects for conceptual development and understanding of pi. Also, PSTs were asked to explain their mathematical ideas during small group presentations and class discussions. In doing so, PSTs ensured that there is relational understanding among themselves during performing micro-teaching. For instance, Group 2 presenter demonstrates:

G2P: Today we have another session, in each group, there are some objects which are provided. Let me ask you one question; discuss and identify objects which are circular.

Group 1: We have stems of cassava, sunflowers and stems of pawpaw.

G2P: For group 1, why did you discover that those are circular objects?

Group 1: Because each object has a circumference and diameter. (Transcribed data).

From the above quote, G2P asked preservice teachers in small group to identify circular objects from non-circular. G2P extended the question by asking PSTs in small groups to explain why the identified objects are circular. Group 1 members responded to the question precisely that the objects are circular because they have circumferences and diameters. Because of that, PSTs had a relational understanding by being able to offer explanations for objects to be called circular.

A relational understanding could not only end up in the research meetings during small group presentations and class discussions, it extended in the reflective journal. There was individual reflection at the end of the daylong meeting. Individual PSTs were asked to reflect on the mathematical ideas that they learned in the research meetings. For instance, the participant addresses:

> What I have understood today about pi is that before, I was knowing that you can get the value of pi only by taking the ratio of the circumference and the diameter. But now, I understand that you can get the value of pi by using the concept radian and practical way by counting the number of diameters required to complete the circumference of a circular object, which gives the value of pi. (Reflection)

From the above quote, it shows that pre-service teacher has a limited understanding on how to find the value of pi. This teacher thought that the value of pi can only be obtained from the ratio of circumference and diameter of a circle. But through participating in the research meetings, this teacher understood multiple ways of finding the value of pi, including counting the number of diameters that goes around the circumference of a circular object. This teacher also offered explanations that the counting process of the number of diameters required to complete the circumference of the object gives the value of pi.

3.5. *PSTs learned how to develop the meaning of pi and its value*

Before participating in four-day research meetings, PSTs thought that pi is a value which is a constant. They also insisted that there is no need to develop its meaning because its value is a known constant. To change this notion, for instance, I asked PSTs on Day 2 to work on the following items to develop the meaning of pi and its value.

i. Measure the diameter of a circular object using a set of vernier callipers. Record the answer on the manila sheet provided. What is the best way for measuring the diameter of the circular object? Explain your answer.
ii. Measure the circumference of a drawn circular object on a plane paper or manila sheet using a string and a set of vernier callipers. Record the answer on the sheet provided.
iii. Find the ratio of the circumference with that of a diameter. Record your answer on the sheet provided. What do you notice(s)? What does this ratio imply? Explain your answer (s).

PSTs worked on the above items in small groups followed by class discussions. Different circular objects were provided in small groups. The circular objects were local materials, including stems of pawpaw (Figure 1), head of a sunflower (Figure 2), and stems of maize (Figure 3).

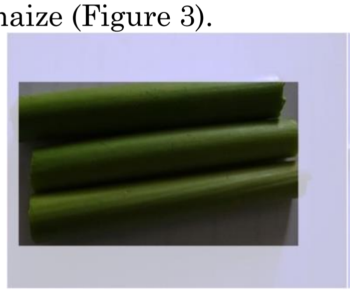
Figure 1


Figure 2


Figure 3

From the above activities and figures, PSTs developing the meaning of pi, through measuring the circumference of a circular object using a string and the diameter of the same object using a set of Vernier calipers. They recorded the two measurements. After that, they computed the ratio of the two measures. They found that the ratio of two measures was equivalent to 3.14. In doing so, they realized that the ratio of the two measures (circumference and diameter) gives the value of pi as an irrational number. Also, by taking the ratio of circumference and diameter of a circular figure/object gives the meaning of pi. As such, real objects helped PSTs realize that pi is a ratio of circumference to diameter of a circular object/figure and the computed value from this ratio is what we call the value of pi. This understanding is evidenced by considering an example below from Group 1 discussions.

According to our discussions, pi is a ratio of the circumference and diameter of a circle. According to the given circular object, it contains the circumference and diameter. And you can measure its diameter and circumference. Diameter is the line dividing the circular object into two equal parts. Now when you take the circumference divides by diameter you get the value of pi, which is a constant value. So, each circle/circular object/circular figure has a constant value which is $\pi \approx 3.14$. So, where c is the circumference of a circle/circular object/circular figure and d is the diameter of the circle/circular object/circular figure. That is, $\pi = \frac{c}{d}$. (Transcribed Data)

3.6. Real objects helped PSTs eliminate systematic errors

Before participating in the four-day research meetings, PSTs thought that the value of pi is $\frac{22}{7}$ and it is always given. However, after working on various items on Day 1, 2 and 3, PSTs realized that the value of pi is not $\frac{22}{7}$; it is an irrational number $(3.14 \ldots .)$. This systematic error was eliminated through using real objects for conceptual development and understanding of pi and its value in the research meetings. This elimination can be evidenced in example below during class discussions.

Facilitator: Is it true that the value of pi is always given?

All participants: Yes

Facilitator: I think you could say, it can be given because there are multiple ways of finding the value of pi, by taking the ratio of the circumference of a circle to its diameter. Also, using those real objects given there, you can get the value of pi by taking the ratio of the circumference of a circular object to its diameter.

All participants: Yes.

Participant 1: From now we have to know that the value of pi is not $\frac{22}{7}$ and it is not always given because it can be computed.

Facilitator: When I ask you to find the value of pi using a stem of sugarcane, will you find it?

All participants: Yes, we can find and get the value of pi. (Transcribed data)

After the above class discussions, I asked PSTs in small groups to work on the activity of finding the value of pi using a stem of sugarcane. For instance, Group 2 presents the procedures and steps to follow when finding the value of pi using a circular object:

i. You have to collect circular objects. For example, in our group, we used a stem of sugar cane as a circular object.

ii. Round the stem of sugarcane using a string by identifying the starting and ending points.

iii. Measure the length of the string by using a ruler. The length of the string gives the circumference of the stem of sugarcane. For example, for our discussions, we measured the length of the string and obtained 10.7cm, which is the circumference of a stem of sugarcane.

iv. Measure the diameter of a stem of sugarcane by using a set of vernier calipers. For example, for our discussions, we obtained 3.4 cm.
v. Obtain the value of pi of a stem of sugarcane. Take the circumference of a stem of sugarcane and divide by its diameter. So, the value of $pi = c/d = 10.7cm/3.4cm = 3.147084 \approx 3.147$ (to 3 decimal places).

(Group learning notes)

From above quote, PSTs developed their understanding of the value of pi through using real objects. PSTs realized that the value of pi is not always given. The value can be computed from circular object/circular figure/circle.

4. Conclusions

Based on the findings in this paper, real objects are important for teaching and learning mathematics. They help students relate mathematics with real objects available in their daily environment. They engage students with mathematics because students learn mathematics by doing. They help students realize that mathematics is not abstract and there are multiple ways of coming to understand a concept. They help students learn mathematics relationally rather than rote.

In this paper I have tried to reveal the importance of using real objects in teaching and learning mathematics by looking at conceptual development and understanding of pi. In the process of exploring the importance of real objects, I paid special attention to real objects available in pre-service teachers' daily environment. The result of this focus was to equip PSTs with knowledge and skills on how to design and implement the lesson in mathematics classes using real objects available in their local environment. In representing the nature of real objects, all natural and artificial objects are important for conceptual development and understanding of a mathematical concept.

This attempt of using real objects, furthermore, can be seen as a contribution to extend understanding of a mathematical concept from enactive to iconic and finally to symbolic representations (Bruner, 1966; Davis & Renert, 2014). For instance, using real objects, such as a stem of pawpaw, PSTs learned how to develop the meaning of pi and its value by measuring the circumference and diameter of the object. They also learned how to draw a cylindrical figure because a stem of pawpaw is a cylindrical shape. They went further by realizing that pi and its value is embedded in a circular part of a cylindrical figure because they were able to measure its circumference and diameter. The value of the ratio of the two measures (circumference and diameter) of the cylindrical figures gave the value of pi and symbolically represented as: $\pi = \frac{C}{D}$ where C represents the circumference, D is the diameter and π represents the pi and its value.

Understanding what real objects and what we want to use in teaching and learning of a mathematical concept is important, given the nature of the learning environment, teaching and learning objectives, nature of the learners, nature of the topic/concept, and teaching and learning outcomes. Adopting a social constructivist approach (Vygotsky, 1978) to teaching and learning mathematics enabled a targeted focus on using real objects for conceptual development and understanding of pi.

Focusing on real objects for conceptual development and understanding of a mathematical concept, how to use real objects available in learners' daily environment is particularly important, given what powerful real objects are to represent for a relational understanding of a mathematical concept. How to use real objects in mathematics classrooms for conceptual development and understanding of a mathematical concept is useful for teachers, educators and textbook developers. For instance, mathematics is often presented in mathematics classrooms as abstract and symbolic. Also, the method of lesson delivery is usually not friendly to the learners. This view affects learners' access to mathematics. Perhaps using real objects can help learners to think mathematically on objects through learning by doing such as in small group discussions, demonstrations, class discussions, and small group presentations. This offers opportunity for the issues of equity in mathematics (see for example, Adler & Pillay, 2017). The better we understand how learning by doing stimulates learning in mathematics classrooms, the better we can design teaching and learning materials to be more inclusive, that allows learners to use real objects to generate mathematical ideas in small group discussions followed by small group presentations and class discussions.

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