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Metacognitive behaviours of eighth grade students with different mathematics achievement levels during problem-solving process

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Abstract

The research explores the metacognitive behaviours demonstrated by secondary school students with different mathematical achievement levels (outstanding, high, moderate and low levels of achievement) during solving a mathematical problem. The research uses a case study method and involves the participation of 36 students. The data, which were collected with the multi-method interview technique, were analysed content analysis. Considering the analyses, it was found that the students with outstanding achievement levels demonstrated more behaviours in number, by spending more time during the problem-solving process; on the other hand, students with low achievement levels demonstrated less behaviour in number by spending less time compared with the other groups. It was found that the students with outstanding, high and moderate levels of achievement demonstrated evaluation behaviours most; in contrast, students with low achievement levels had the behaviours of awareness the most during the problem-solving process. In addition, it was concluded that the groups started the process generally with a behaviour in the dimension of awareness, used the behaviours of regulation and finished the process with behaviour in the dimension of evaluation. The teacher should guide students by focusing on some behaviours respectively related to the dimensions of awareness, regulation and evaluation during solving a problem in the course.

Keywords: Mathematical problem; problem-solving; metacognition; metacognition in problem-solving

1. Introduction

Problem-solving can be defined as the individual's thinking process to obtain new knowledge about the situation until s/he overcomes the tension of the problem and to search for a logic appropriate to the problem situation by applying his/her mathematical knowledge (Lester & Kehle, 2003, p.505). The concept of problem-solving refers to a

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process (Polya, 1973; Zawojewski & Lesh, 2003). This process contributes to the competence of using mathematical skills and applying these skills in real-life situations (Abu-Elwan, 1999) and is specifically emphasised in curricula of mathematics teaching institutions (NCTM, 1989, 1991, 1995, 2000) and the curricula of national education (The Ministry of National Education [MoNE], 2018). Owing to the significance that problem-solving has a relationship with mathematical achievement (Bhat, 2014; Özsoy, 2005), it is crucial to determine its significance and all the variables that may influence it and its effect levels. In this scope, there are research results in the literature suggesting that this may influence the students' problem-solving achievements in terms of the variables such as self-efficacy (Hackett & Betz, 1989), mathematics anxiety (Cooper & Robinson, 1991), self-regulation (Pintrich & De Groot, 1990), mathematical belief and learned helplessness related to mathematics (Ağaç & Masal, 2017), gender, attitude (Özgen et al., 2017; Uysal 2007), number perception (Işık & Kar, 2011) and metacognitive skills/behaviours (Author, 2016; Desoete et al., 2001; Lesh, 1982, 1985; Lester, 1980, 1983, 1985; Lester, Garofalo & Kroll, 1989; Kazemi, Fadae & Bayat, 2010; Mayer, 1998; Schoenfeld, 1992; Silver 1985; Sweeney, 2010 etc.).

Among these variables, we focused on the metacognitive skills/behaviours in this research. The metacognition concept can be defined as the individuals' regulating the information about the cognitive behaviours and these behaviours during the learning processes (Brown, 1978; Flavell, 1976). It will be insufficient to deal with the problem-solving process with only its cognitive dimension (Artzt & Armour-Thomas, 1992; Wilson, & Clarke, 2004, Yong & Kiong, 2006), but metacognition needs to be evaluated as a significant variable in the problem-solving process considering the necessity of the sufficient level of awareness related to this knowledge, in addition to the cognitive knowledge in solving process, (Garofalo & Lester, 1985) and checking, planning, monitoring and evaluation for the solution process (Garofalo &Lester, 1985; Kaur, 1997; Wong, 1992; Yong & Kiong, 2006). There are some results in the literature suggesting that the students, who demonstrate metacognitive behaviours frequently, are more successful in the problem-solving process (Şengül & Yıldız, 2013; Yimer & Ellerton, 2010), besides, that they are more successful in problem-solving (Aydemir & Kubanç, 2014; Panaoura, Philippou & Christou, 2003; Şengül & Katrancı, 2015; Yimer & Ellerton, 2010; Yong & Kiong, 2006). In addition, the education related to the metacognitive behaviours may provide an increase in mathematical problem-solving achievements of students (Adibnia & Putt, 1998; Desoete, Roeyers & De Clercq, 2003; Kramarski, Mevarech & Arami, 2002; Mevarech, 1999; Mevarech & Kramarski, 2003; Nancarrow, 2004; Özsoy & Ataman, 2009; Verschaffel, 1999; Verschaffel et al., 2009).

Briefly, the problem-solving skills are significant for mathematical achievement, and the metacognitive skills/behaviours for the problem-solving skills. Accordingly, the research results, suggest that the metacognitive activities of the students with relatively low mathematics achievement, are also low in the mathematical problem-solving process

(Abdullah, Rahman & Hamzah, 2017; Desoete et al., 2001; Kuzle, 2018; Sweeney, 2010), can be given as the example for this relationship. However, the point that is remarkable here is that demonstrating the metacognitive behaviours frequently does not guarantee the achievement in the problem-solving process (Author, 2016; Wilson & Clarke, 2004). Considering this, it is crucial to investigate the metacognitive behaviours that the students at the high mathematics achievement levels demonstrate in the solving process of a problem and their sequence. Nevertheless, it is necessary to take the difficulty of the measurement of metacognition into account. To overcome the relevant difficulty, the MMI method, which was developed by Wilson and Clark (2004), and has been described in the Data Collection section of this study in detail, was employed in this research. Within the scope of this method, the metacognitive behaviours were evaluated under three categories as awareness, evaluation and regulation. These categories can be defined as;

- Awareness: "Metacognitive awareness relates to individuals' awareness of where they are in the learning process or in the process of solving a problem, of their content-specific knowledge, and their knowledge about their learning or problem-solving strategies",
- Evaluation: "Metacognitive evaluation refers to judgements made regarding one's thinking processes, capacities and limitations as these are employed in a particular situation or as self-attributes",
- Regulation: "Metacognitive regulation occurs when individuals make use of their metacognitive skills to direct their knowledge and thinking" (Wilson & Clark, 2004).

There are studies, conducted with similar purpose, on the second (Kuzle, 2018; 2019); third (Desoete et al., 2001); fourth (Kuzle, 2018), fifth (Fazira, Sukoriyanto & Rahardjo, 2020), sixth (Wilson, & Clarke, 2004) class levels and with the pre-service mathematics teachers (Barbacena & Sy, 2015; Author, 2016). Considering that the metacognition concept may differ depending on students' ages, strategies and knowledge levels about the topics (Brown 1978; Desoete et al., 2001; Veenman, Van Hout-Wolters & Afflerbach, 2006; Panaoura et al., 2003; Senemoğlu, 2007) and the structure of the problem (Mokos & Kafoussi, 2013), the common focuses in the results of the relevant researches can be summarised as follows.

- The process starts mostly with a behaviour in the dimension of awareness (Author, 2016; Fazira et al., 2020; Kuzle, 2018, 2019; Wilson & Clarke, 2004).
- Although the awareness dimension is the behaviour dimension that is used most (Kuzle, 2018 (at the 2nd class level)); on the contrary, there are also studies (Author, 2016; Kuzle, 2018 (at the 4th class level); Wilson & Clarke, 2004) suggesting that it is the least used dimension. The most used in number in the

relevant literature is the deficiencies of students' mathematical problem-solving culture (Kuzle, 2018); the least use in number in awareness is metacognitive activity as it is not recognised by the student due to its less distinctive characteristics (in the sense of conscious recall) compared with evaluation and regulation (Wilson & Clarke, 2004.

- Evaluation is the most used metacognitive behaviour dimension (Barbacena & Sy, 2015; Author, 2016; Kuzle, 2018, 2019; Wilson & Clarke, 2004).
- The problem-solving process generally finishes with a behaviour in the dimension of evaluation (Author, 2016; Fazira et al., 2020; Kuzle, 2018, 2019; Wilson & Clarke, 2004). This situation was evaluated as the result of the result-oriented mathematics curricula rather than the process (Fazira et al., 2020; Kuzle, 2018, 2019).
- Regulation is the least used behaviour dimension in number (Kuzle, 2018, 2019).
- There is no linear structure in the metacognitive behaviour sequence in the problem-solving process (Author, 2016; Kuzle, 2018, 2019; Wilson & Clarke, 2004).
- In a successful problem-solving process, there are short (Wilson & Clarke, 2004) or long (Author, 2016; Kuzle, 2018, 2019) contrasting results for using metacognitive behavioural sequences.
- The length of the metacognitive behaviour sequences may increase even according to the participants' metacognitive development levels (Desoete et al., 2001; Kuzle, 2018, 2019;)
- It was realised that a high level of metacognitive awareness is also crucial in terms of reaching the correct results or not reaching the incorrect results in the problem-solving process (Barbacena & Sy, 2015; Author, 2016).

As has been mentioned, the researches, which were carried out with different class levels in this topic, have presented significant information in terms of determining the metacognitive behaviours that the students demonstrate in the problem-solving process. Conducting similar research from each class level is useful for the literature to have a stronger background in this direction. From this point, it is aimed to explore the metacognitive behaviours that the eighth-class students with different mathematical achievement levels (outstanding, high, moderate and low achievement) demonstrate in solving a mathematical problem. Considering the results that will be obtained in this study and the behaviours of the students at each achievement level, it is thought that it will provide more efficient clues to the instructors for the attempts to develop the metacognitive skills of students.

2. Method

This research uses a case study method. The case study is the research method, which enables researchers to examine one or more events in a limited time with the data collection tools consisting of multiple resources (Creswell, 2007). The case study method was applied to explore the behaviours of the students and how they carry out the process during problem-solving.

2.1. Study group

The research was carried out with the 36 students at the eighth-class level. The mathematics course academic achievements of the students were taken into consideration in selecting the participants. The study group was formed in two stages. In the first stage, the school and institution, in which the study would be carried out, were selected with a convenience sampling method as the school in which one of the participants was in-service, specifically to minimise the loss of time and labour (Saumure & Given, 2008). One of the schools that the researcher investigated, was the Science and Art Centre, which was the only one in the province. This centre gives education to the students who have a higher capacity of intelligence, creativity, art, management or in specific areas compared with their peers. The instructors, who are experts in their profession, educate the students who have outstanding achievement levels and can be categorised as gifted. The other school, which is in the centre of the province, is one of the five schools at the top with the highest achievement scores in the LGS (The exam that students have to take to enter the high schools in Turkey) in the province. In the second stage, a criterion sampling technique, which means including materials or persons in the study based on previously defined criteria (Patton, 1990), was used and students' achievements in mathematics courses were determined as the criterion. In this stage, the students were grouped with 'low', 'moderate' 'high' and 'outstanding' achievement levels. The achievement levels were specified considering the in-class performances consisting of the students' achievement scores, mathematics achievement scores and teacher opinion. Based on the grading system in Turkey, the students with the average score between 0-45 were considered as "low" achievement level; 45-70 as "moderate" and 70 and above as "high". The students in the group of outstanding achievement levels were determined from all the students in the field of general skills in the science and art centre. As the Ministry of National Education selects these students from the field of general skills with the exam and verbal interview, they are regarded as good problem solvers. Since there were 9 students in the eighth class in the science and art centre, the number of students in the other groups were selected as 9 to be equal to this group.

2.2. Data collection tools

The data were collected with a Multi-Method Interview technique developed by Wilson (2001) in the research. In this scope, the mathematical problem was specified first. A

problem pool was formed by reviewing the literature in determining the problem. The problems in the pool were evaluated according to the expert opinions and the most appropriate one was selected. The problem, which was selected, is as follows.

> *Three shepherds decide to eat something. They will share the cost among themselves. The first shepherd brings 5 food containers, the second one 3 containers. However, the third one does not bring food. After the meal, the third shepherd takes 8 silver coins from his pocket and gives 5 to the first and 3 o the second shepherd. However, the first one protests this and argues that he deserves more; this will not be equal sharing. Arguing for a while, they reach the correct result. How many coins should the third shepherd pay to the others or how should they share the 8 coins to ensure equal sharing?*

This problem was selected as it consisted of various ways of solutions and rich components in terms of gathering information. In addition, it was tried to select as a non-routine problem that students were less familiar with. Because, a problem, which students are not familiar with, provides an opportunity to reveal the metacognitive behaviours more in number (Şengül & Yıldız, 2013). In addition, the factors that affected perceptions of problem difficulty such as; categorisation, interpretation, resource relevance, and complexity were also considered (McGinn & Boote, 2003). The problem, that was selected, was asked to the four students who did not participate in our research. In the pre-application, it was found that all the behaviours that the participants demonstrated during the process were included in the behaviour cards; according to the evaluation of the research problem, the problem was realised to be suitable for the research.

2.3. Data collection process

The students were asked to solve the problem by thinking aloud with the multi-method interview technique. In this process, which was recorded with a video camera, the students tried to solve the problem by thinking aloud. After the students finished the problem-solving process with thinking aloud, it was tried to find out which metacognitive behaviours were applied in this process. For this, the metacognitive behaviour cards, which had been formerly developed, was used (Wilson, 2001). The dimensions of components of the metacognitive behaviour cards are presented below.

Awareness: I thought about what I already know. (A1)

I tried to remember if I had ever done a problem like this before. (A2)

I thought about something I had done another time that had been helpful. (A3)

I thought 'I know what to do. (A4)

I thought 'I know this sort of problem. (A5)

Evaluation: I thought about how I was going. (E1)

I thought about whether what I was doing was working. (E2)

I checked my work. (E3)

I thought 'Is this right? (E4)

I thought 'I can't do it. (E5)

Regulation: I made a plan to work it out. (R1)

I thought about a different way to solve the problem. (R2)

I thought about what I would do next. (R3)

I changed the way I was working. (R4)

The metacognitive behaviour cards, developed by Wilson (2001), were formed as three basic dimensions. Two experts, one of whom was in the field of English education and one was in the field of metacognition, translated the behaviour cards jointly. At the end of the problem-solving process, the students were asked to express the metacognitive behaviours that they demonstrated during the problem-solving process using the behaviour cards in order. Blank cognitive behaviour cards, which the students would write their behaviours on to express their arithmetic operations, were given to them in this process. The students had the opportunity to use any of the behaviour cards more than once. After they had put the cognitive and metacognitive behaviour cards in order, the interview was completed. Then, the students watched the interview process which had been recorded, and they were asked to confirm the sequence of the metacognitive cards related to the problem-solving process. During the confirmation process, an exchange of ideas between the researcher and student was held related to what s/he thought at the relevant period. The final sequences of the participants, who wanted to change the order of the metacognitive behaviour cards, were taken into consideration. The problem solving and metacognitive behaviours determination/classification process of each student was transcribed in its deepest detail, divided into periods and written. An example of the solution process is presented in Table 1.

Table 1. *Sample transcript*

Duration	Observed Behaviour
0:00 - 0:32	The student read the question aloud.
0:33 - 1:53	S/he expressed the question with his/her sentences by rereading from the beginning. "Normally 5 coins for 5 containers, 3 coins for 3," said s/he. Then, s/he started to reread the question from the middle. S/he remained silent for a while.
1:54 - 2:49	S/he read the question again. Then s/he wrote 5 containers and 5 coins to the opposite; 3 containers and 3 coins. S/he started to read the question again.

2:50 - 3:31	Keeping silent for a while, s/he said, "I think if I should give equal to both of them"After thinking for a while, s/he said, "Why the first one objects?" Then, replied, "He brings more food than the other and gets more money, Sir" and thinking for a while, added, "so, the first one requests more than 5 coins".
3:32 - 3:44	S/he kept thinking of looking at the problem. S/he said, "the second one does not object to this case." After reading the question for a short, added: "I wonder if he will give 8 to 0".
3:45 - 4:41	After thinking for a while, s/he wrote 1 and 2 on the paper, then, wrote x under 1 and y under 2. Under these expressions, s/he wrote $x > 5$ and $y < 3$ and started to read the problem again.
4:42 - 5:08	During expressing the problem with his/her sentences, s/he read the part of 5 coins and 3 coins again. Thinking for a short, s/he started to read the question again. Then, s/he continued to interpret the problem with his/her sentences.
5:09 - 5:38	During expressing the problem with her sentences, suddenly s/he said, "I want to say 7 and 1 Sir." Writing 1st and 2nd on the paper, s/he wrote 7 and 1 in order and added, "I thought 7 coins for the first shepherd, 1 coin for the second."
5:39 - 6:29	After writing the solution and expressing it, s/he said, "As the first one objects, he wants more than 5 coins. He can get 6, 7 and 8. However, I do not think he will give 0 for the second. It should be 6 to 2 or 7 to 1." Then, s/he added, "He may have wanted to increase the number of coins as much as the containers, so I want to say 7 to 1" and completed the solution.

As an example, is presented in Table 1, these transcripts were used to determine the problem-solving duration of the participants and their behaviours in the process.

2.4. Analysis of the data

In the analysis of the data, we applied content analysis techniques to investigate the relationship between the problem-solving process and metacognition in depth. The data were analysed using the content analysis method, which systematically decomposes for specific purposes (Merriam & Grenier, 2019).

During the coding process, the awareness, evaluation and regulation dimensions, which were taken as the base related to the form of metacognition by Wilson (2001), were used as the categories; the metacognitive behaviours written on the 14 behaviour cards within the scope of the mentioned dimensions were used as the codes. In addition, the students' behaviours such as the arithmetic operations, mathematical comparisons except for the metacognitive behaviours of the students, written on the papers by them were coded as the cognitive behaviours.

The data of the research were coded simultaneously by the two researchers. During coding the students with four different achievement levels, K1 code was used for the students' group with outstanding achievement level, K2 code for those with high achievement level, K3 code for those with moderate achievement level and K4 code for those with low achievement level. To code the students individually, the code of the group that the student belonged to was written and continued with the sequence number of the

relevant group. For instance, as the K2 code was used for the student group with a high achievement level, the K.2.1 code was used for the first student in this group. Thus, the data were analysed by coding all the participants with the group number and the sequence number in the group. The gathered findings are presented with tables and graphs below.

3. Findings

The findings related to the sub-problems of the research are presented in this section. The dimensions related to the behaviours demonstrated by the students during the problem-solving process were coded as awareness "A", evaluation "E", regulation "R" and cognitive "C".

3.1. Findings related to the students group with outstanding achievement (K.1.)

The metacognitive behaviours demonstrated by the students with outstanding achievement levels, their problem reading and solving time are presented in Table 2.

Table 2. *Data related to the problem-solving process of the students in the group of outstanding achievement*

Participants	*Metacognitive Behaviours Demonstrated by the Participants*	*Number of Problem Reading Aloud*	*Problem Solving Time*
K.1.1.	A_4-C- E_1- C-R_4-R_1-C-R_2-R_1-C-E_5-E_5	*6*	*7,21*
K.1.2.	E_4-A_1-C-E_2-E_1-E_4-A_1-C-R_4-C-R_1-E_2	*5*	*4,51*
K.1.3.	A_2-A_5-A_4-C-R_2-E_3-C-R_1-R_4-C-E_2-E_4-E_3-E_1	*5*	*5,46*
K.1.4.	R_3-A_1-C-A_2-A_3-R_2-R_3-C-E_1-C-E_3-E_2-A_1-R_1-E_3-E_2-E_5	*2*	*4,29*
K.1.5.	A_1-C-E_2-R_2-A_2-C-E_3-R_1-E_1-E_4-E_3-R_4	*4*	*4,22*
K.1.6.	E_4-A_2-R_3-A_1-C-E_5-E_3-C-C-E_4-A_1-R_2-E_2-R_4-E_3-E_1-E_3-E_4	*3*	*4,44*
K.1.7.	A_5-A_2-C-R_3-A_1-E_1-C-E_2-E_3-R_4-E_5-R_2-E_4	*6*	*7,30*
K.1.8.	A_3-A_2-R_3-C-E_2-E_3-E_1-C-R_1-R_4-C-A_3-C-E_4	*8*	*14,21*
K.1.9.	A_1-A_1-C-R_3-A_4-E_2-C-E_1-R_2-R_4-R_2-C-E_3-E_5	*5*	*6,29*

The behaviours that the participants demonstrated are the abbreviations as A "awareness", E "evaluation", C "regulation" and the number in the bottom index refers to the behaviour card presented in the Data Collection section.

As it is presented in Table 2, among the 127 behaviours demonstrated by 9 participants in the K.1 group, 24 of them are related to the dimension of awareness, 46 to the evaluation, 29 to the regulation and 28 to the cognitive behaviour. When these behaviours are analysed, it is noticed that all of the participants take notes of the information given in the question on the paper in the dimension of behaviour. In the awareness dimension, most of the participants demonstrated the behaviour of "I thought of what I already know about the problem". In the dimension of regulation, except for the K.1.3 coded participant, all of the participants applied the behaviour of "I thought about a different way to solve the problem", during the problem-solving process. Almost all of the participants in the K.1 group demonstrated the behaviours in the dimension of

evaluation. The K.1.1 coded participant demonstrated the behaviour in the regulation dimension most in number. K.1.9 coded participant demonstrated the behaviours in the evaluation and regulation dimensions equally. Other participants demonstrated the behaviours in the evaluation dimension most in number. All of the participants in the evaluation dimension demonstrated the behaviour of "I thought about whether what I was doing was working." In addition, all of the participants except for the K.1.1 coded participant used the behaviour of "I checked my work." Another behaviour demonstrated in the dimension of evaluation most in number is "I thought 'Is this right?". Other participants except for the K.1.1 and K.1.2 coded participants demonstrated this behaviour.

In addition, the participants in this group spent 59 minutes 53 seconds in problem-solving. The behaviour cycles of the students with outstanding achievement levels are presented in Figure 1.

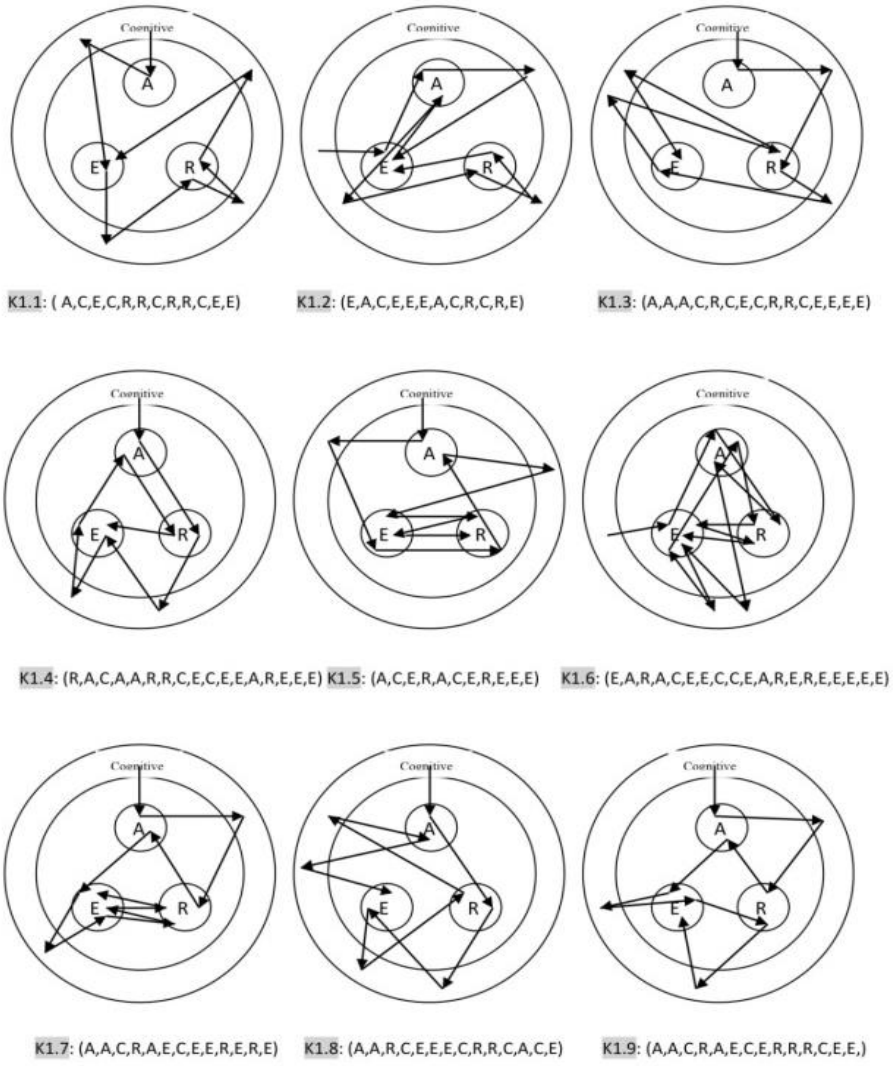

Figure 1. Behaviour order of the students with outstanding achievement

As it is presented in Figure 1, when the order of dimensions of the behaviours demonstrated by the participants in the group of K.1 is taken into consideration, it is seen that there is no stable transition between dimensions in the order of behavioural dimensions, and all participants make reversible transitions between the dimensions. In addition, most of the participants in this group started problem-solving demonstrating a behaviour in the dimension of awareness and completed the process with behaviour in the evaluation dimension. The most demonstrated behaviours of the participants in this group during the process in the dimension of the evaluation were “I thought 'Is this right? and “I thought 'I can't do it.”

3.2. Findings Related to the Students Group with High Achievement (K.2.)

The metacognitive behaviours, problem reading and solving time of the students with high achievement levels during the problem-solving process are presented in Table 3 below.

Table 3. *Data related to the problem-solving process of the students in the high achievement group*

Participants	*Metacognitive Behaviours Demonstrated by the Participants*	*Number of Reading Problem Aloud*	*Problem Solving Time*
K.2.1.	A_3-E_4-R_3-R_2-E_1-C-E_3-C E_5-R_1-R_2-E_1-C-E_2-E_3-E_5	4	9,11
K.2.2.	A_1-A_4-E_3-E_4-A_4-E_4-A_2-E_1-R_4	7	5,58
K.2.3.	A_2-A_1-C-R_3-R_2-A_1-C-R_4-R_1-C-R_4-R_1-E_1-E_4	4	6,31
K.2.4.	A_1-A_2-C-R_4-C-E_2-E_4	5	4,58
K.2.5.	A_4-A_1-R_2-A_3-R_3-A_4-R_4-E_2-E_3-C-E_5-E_4	1	1,43
K.2.6.	E_4- A_2- R_4- R_2-C	4	2,4
K.2.7.	E_4-A_2-A_1-R_4-C-E_3-E_2-E_1-E_4	5	3,52
K.2.8.	A_1-R_3-E_4-A_2-C-C-E_3-E_1-R_2-E_2-E_5	4	2,24
K.2.9.	E_4-A_2-A_1-C-C-E_2-R_4-C-C-C-E_1-E_3-E_5-E_4	2	5,05

The behaviours that the participants demonstrated are the short forms as A "awareness", E "evaluation", C "regulation" and the number in the bottom index refers to the behaviour card presented in the Data Collection section.

As it is presented in Table 3, 21 of the 97 behaviours of the nine participants in the K.2 group are in the dimension of awareness, 40 of them in evaluation, 20 regulation and 18 cognitive dimensions. As it is analysed with the separate evaluation of the participants, it is seen that the K.2.3 and K.2.6 demonstrated the behaviours in the regulation dimension most in number; on the other hand, other participants demonstrated the behaviours in the dimension of evaluation most in number. All of the participants used the behaviour card "I thought 'I can't do it." in the dimension of evaluation. In addition, other participants except for the K.2.2, K.2.3 and K.2.6 participants used the card "I thought about how I was going." in the solution process.

The time that the participants in the K2 group spent during the problem-solving process is 42 minutes 22 seconds. While K.2.1 was the participant who spent the longest time as 9 minutes and 11 seconds in the problem-solving process, K.4.5 was the one who spent the shortest time with the problem spending 1 minute 43 seconds in this group. The behaviour cycles of the students with high achievement levels are presented in Figure 2.

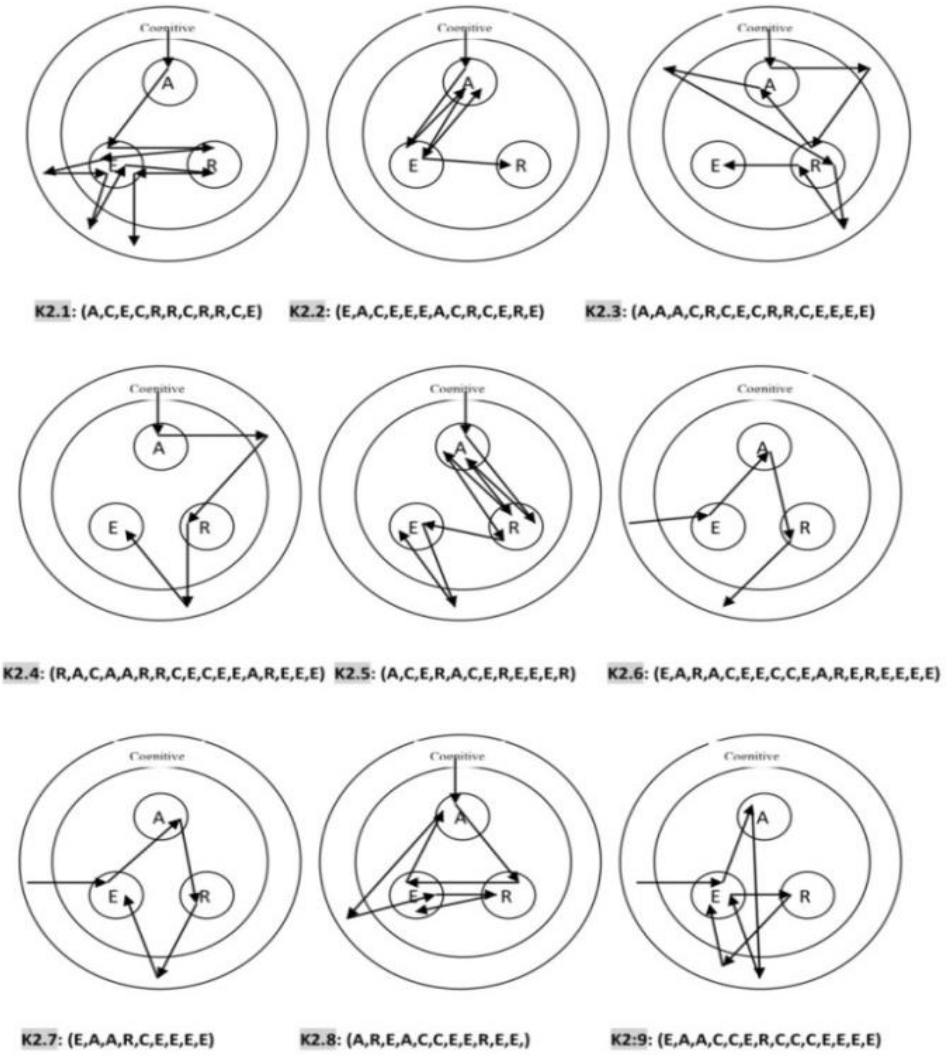

Figure 2. Behaviour order of the students with high achievement

As it is seen in Figure 2, there are reversible transitions between the dimensions of the behaviours demonstrated by the participant in the K.2 group. It is noticed that most of the participants in this group started the problem solving with a behaviour in the dimension of awareness and completed the process with a behaviour in the dimension of evaluation. The K.2.7 and K.2.9 coded participants started the process with the "I thought 'I can't do it." thinking skill card in the evaluation dimension and completed the process with the same thinking skill. In the dimension of awareness, it was noticed that the participants started the process with different metacognitive behaviour cards. As the behaviour skill dimensions of the participants in the K.2 group during the process were compared, it is found that they demonstrated at least one behaviour among the dimensions of awareness, evaluation and regulation during the process. In addition, it is

another finding that most of the participants in the cognitive behaviour dimension took notes on the given paper in their second and third reading of the problem.

3.3. Findings Related to the Students Group with Moderate Achievement (K.3.)

The metacognitive behaviours of the students with moderate achievement levels during solving the problem, reading the problem and solving time are presented in Table 4.

Table 4. Data related to the problem-solving process of the students with moderate achievement level

Participants	*Metacognitive Behaviours Demonstrated by the Participants*	*Number of Reading Problem Aloud*	*Problem Solving Time*
K.3.1.	A_4-R_2-R_3-C-A_5-A_1-A_1-R_4-E_2-R_1-C-E_1-A_3	4	5,39
K.3.2.	E_4-A_1-A_4-E_1-E_2-E_1-R_3-E_3-C-E_5-A_2-A_3	2	2,43
K.3.3.	E_4-A_1-R_4-E_4-R_2-R_3-E_1-C-E_5	3	2,46
K.3.4.	A_1-R_2-C-A_2-R_3-R_2-E_1-E_4-E_2-E_3-E_4	5	3,45
K.3.5.	A_1-R_2-C-E_1-E_2-R_4-R_3-A_2-E_1	3	3,44
K.3.6.	A_1-A_4-R_1-R_2-C-E_4-R_4-A_2-R_2-E_3-R_1-A_1-A_5-C-C-E_3-E_2	2	2,33
K.3.7.	A_2-R_2-A_1-A_4-R_4-A_3-R_1-A_1-E_2-E_1-R_1-A_5-E_4	2	1,58
K.3.8.	R_3-A_1-A_4-C-E_2-A_3-A_5-E_1-E_3	2	1,47
K.3.9.	E_1- E_2- E_3- A_1- A_5- A_2- R_4- R_2-C-E_5	5	4,13

The behaviours that the participants demonstrated are the short forms as A "awareness", E "evaluation", C "regulation" and the number in the bottom index refers to the behaviour card presented in the Data Collection section.

As it is presented in Table 4, 104 behaviours were demonstrated in total in the student group with moderate achievement levels. 35 of these behaviours were related to evaluation, 32 awareness, 26 regulation and 11 cognitive behaviour dimensions. Among the behaviours demonstrated in the moderate achievement group, a similar number of behaviours were demonstrated in the dimensions of awareness and evaluation. Except for the K.3.3 participant, all of the others in this group demonstrated the behaviour of "I thought about whether what I was doing was working" in the evaluation dimension. In addition, it was noticed that all the participants except for the K.3.3 and K.3.6 coded participants, demonstrated "I thought about how I was going." behaviour in the evaluation dimension during the process. Besides, it was noticed that all of the participants demonstrated the behaviour of "I thought about what I already know." among the behaviours in the dimension of awareness. As the participants with moderate achievement levels were analysed separately, it was seen that 3 of the participants demonstrated the behaviours in the awareness dimension and 3 in the evaluation dimension most in number. The K.3.5 coded participant demonstrated equal numbers of behaviours in evaluation and regulation dimensions. Besides, the K.3.6 coded participant

demonstrated the behaviours in the dimensions of evaluation, regulation and awareness that were equal in number.

The total time that the students in the group of moderate achievement level spent 29 minutes 8 seconds in the problem-solving process and becomes the participants who spent the longest time in solving the problem, on the other hand, the K.3.7 coded participant spent the shortest time during problem-solving with 1 minute 47 seconds. The behaviour cycles of the students with moderate achievement are presented in Figure 3.

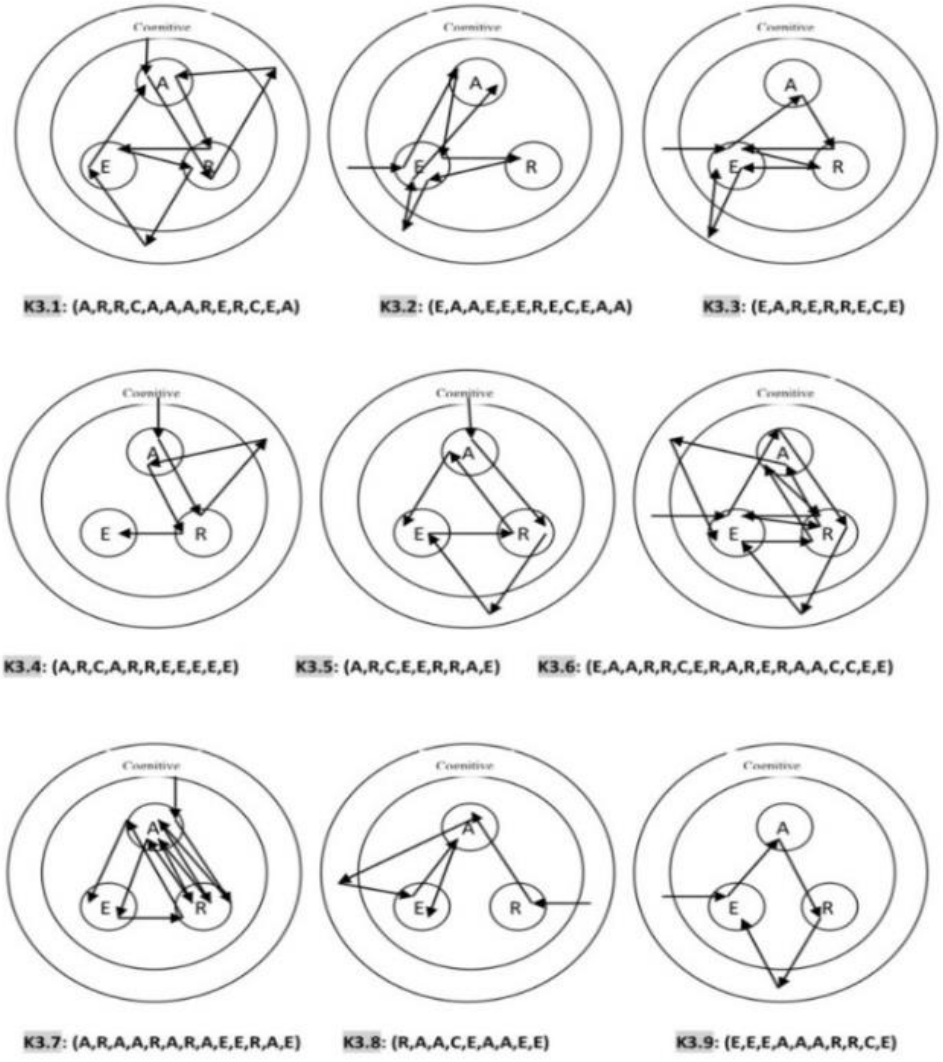

Figure 3. Behaviour order of the students with moderate achievement

As it is indicated in Figure 3, it is seen that there are irreversible transmissions between the behaviour dimensions. As it was considered within the scope of the dimensions, it was found that all the students demonstrated at least one behaviour from all the dimensions. As which behaviour did the participants at the level of moderate achievement demonstrate to start the problem-solving process was analysed, it was found that 4 participants started demonstrating a behaviour in the dimension of awareness, 4 in evaluation and 1 in regulation. 3 participants, who started the process with a behaviour in the dimension of evaluation, started the process with the behaviour of "I thought 'I can't do it." As the last behaviour dimension of the participants in the moderate achievement group was analysed, it was observed that all of the participants except for the K.3.1 and K.3.2 coded participants, demonstrated at least one behaviour in the dimension of evaluation. In addition, K.3.1 and K.3.2 coded participants completed the process with the behaviour of "I thought about something I had done another time that had been helpful." which was in the dimension of awareness.

3.4. Findings Related to the Students Group with Low Achievement (K.4.)

The metacognitive behaviours demonstrated by the students with low achievement, reading and solving time of the problem during the problem-solving process are presented in Table 5.

Table 5. Data related to the problem-solving process of the students with low achievement group

Participants	*Metacognitive Behaviours Demonstrated by the Participants*	*Number of Reading Problem Aloud*	*Problem Solving Time*
K.4.1.	R_3-A_1-E_2-A_5-R_1-A_2-R_3-E_4	2	2,18
K.4.2.	E_4- R_2-E_2-A_2	3	2,42
K.4.3.	A_4-C-R_3-E_2-A_1-R_1-R_4-E_3-R_4-A_3-E_5	4	4,3
K.4.4.	E_4	2	2,06
K.4.5.	A_1-A_4-R_4-C-A_5-E_3-R_2-E_4	2	2,36
K.4.6.	A_2-A_1-E_4-A_5-A_4-E_1-E_2-E_4	3	1,37
K.4.7.	E_4-A_4-A_2-R_2-R_1-A_5-A_1-C-E_3-E_4	4	4,02
K.4.8.	A_5-R_2-E_4-A_4-R_2-A_1-E_2-A_2	3	3,21
K.4.9.	A_2-E_4-C-A_3-E_3-R_4-E_3-R_2-E_2-A_1-E_4	2	2,29

The behaviours that the participants demonstrated are the short forms as A "awareness", E "evaluation", C "regulation" and the number in the bottom index refers to the behaviour card presented in the Data Collection section.

As it is presented in Table 5, participants in the K.4 group demonstrated 69 behaviours in total. When the number of behaviours that were demonstrated by the students in this group was analysed according to the dimensions, it was found that 25 behaviours were demonstrated in the dimension of awareness, 24 in evaluation, 16 in regulation and 4 in cognitive dimensions in number. As the behaviours, which was

demonstrated most related to the awareness dimension, were considered, all the participants, except for the K.4.2 and K.4.4 coded participants, demonstrated the "I thought about what I already know" behaviour. In addition, among the participants in this group, the K.4.4 coded participant, who demonstrated only one behaviour, demonstrated the behaviour of "I thought 'I can't do it" in the dimension of evaluation and completed solving the problem. As the evaluation dimension in this group was analysed, all the participants, except for the K.4.3 participant, demonstrated the behaviour of "I thought 'I can't do it." In addition, the participants except for the K.4.4, K.4.5 and K.4.7 demonstrated the behaviour of "I thought about how I was going" in the dimension of evaluation. As the findings in the dimension of regulation were analysed, it was noticed that most of the participants demonstrated the behaviour of "I thought about what I would do next". Another finding related to this group was that the K.4.4 and K.4.6 coded participants demonstrated no behaviour in the dimension of regulation. Besides, another finding was that the participants in this group demonstrated less cognitive behaviours in number compared with the participants in other groups.

As the time of reading the problem aloud by the K.4 group was analysed, it was noticed that the participants in this group read the problem in 42 seconds. The K.4.2 coded participant, who read the problem very slowly, completed it in 57 seconds. The K.4.1 coded participant, who read the problem quickly, finished reading in 34 seconds. In addition, the participants in this group spent 25 minutes 41 seconds solving the problem. The behaviour cycles of the students with low achievement were presented in Figure 4.

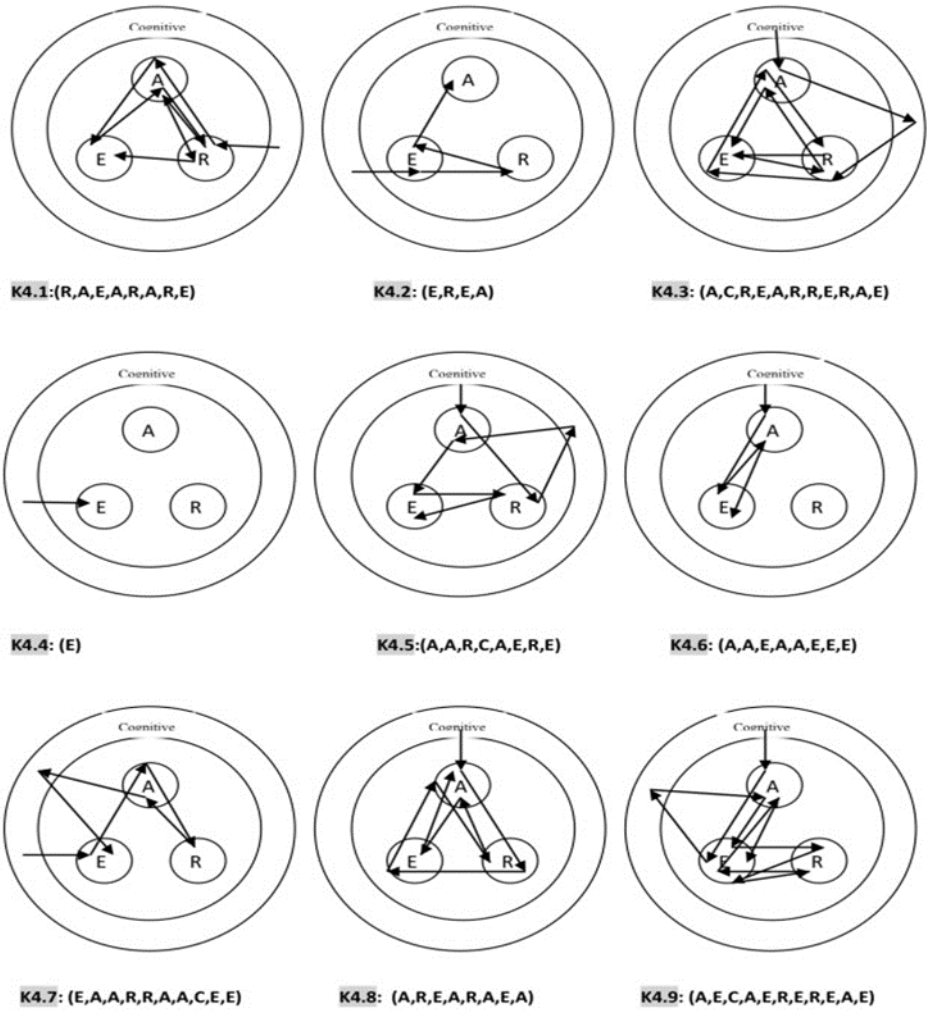

Figure 4. Behaviour order of the students with low achievement

As Figure 4 is analysed, it is seen that there is no stable transition between the dimensions in the order of behavioural dimensions, and all participants make reversible transitions between the dimensions. As which behavioural dimension did the participants in K.4 group start the problem-solving process was analysed, it was seen that 5 of them started with a behaviour in the dimension of awareness, the rest 3 with the behaviours in the evaluation and 1 with the behaviour in the dimension of regulation. The participants, who demonstrated a behaviour in the dimension of awareness, started the process by demonstrating different behaviours. As which behaviour did the participant in the K.4 group demonstrate in completing the process was analysed, the participants, except for the K.4.2 and K.4.8 coded participants, completed the process

with a behaviour in the dimension of evaluation. Most of the participants, who completed the process with the dimension of evaluation, finished the process with the behaviour of "I thought 'I can't do it". Among the participants, who completed the process with the awareness dimension apart from the evaluation dimension, the K.4.2 and K.4.8 coded participants completed the process with the behaviour of "I tried to remember if I had ever done a problem like this before".

4. Results and Discussion

The research results related to the metacognitive behaviours demonstrated by the students with different mathematical achievement levels (outstanding, high, moderate and low levels) during the problem-solving process are presented in this section.

When the total number of behaviours demonstrated by the groups were taken into consideration, it was found that the group that demonstrated the highest number of behaviours was the student group with outstanding achievement. There is a difference that may be regarded as significant between the total number of behaviours that the student group with outstanding achievement and the number of behaviours of other groups. The number of behaviours demonstrated by the students with outstanding achievement is approximately twice more than those of the low achievement group. While the students with high and outstanding achievement levels demonstrated a similar number of behaviours in this process, there was a significant difference between the students with low achievement and those with high and moderate achievement in terms of the problem-solving duration. These results overlap with the research results suggesting that the students with the low level of mathematics achievement demonstrate less metacognitive activities in the problem-solving process (Abdullah et al., 2017; Desoete et al., 2001; Kuzle, 2018; Sweeney, 2010) and the academic achievements of the students with high metacognitive awareness or using the metacognition strategies more are higher than the academic achievements of the students using them less (Jaafar & Ayub, 2010; Mayer, 1998; Ormrod, 2003; Özsoy, 2011; Schunk & Zimmerman, 1998). Similar results suggest that the students with high academic achievement levels even in different disciplines, except for mathematics, demonstrate the metacognitive behaviours more in number compared with the students with low academic achievement (Demir, 2013; Doğanay & Demir, 2011).

When the total number of behaviours' dimensions demonstrated by the groups was considered, it was found that, while the students with outstanding, high and moderate achievement demonstrated most behaviours in total in the dimension of evaluation, the students with low achievement demonstrated behaviours in the dimension of awareness most in number. This result is parallel with the research results suggesting that evaluation is the most applied metacognitive behaviour dimension (Barbacena & Sy, 2015; Author, 2016; Kuzle, 2018, 2019; Wilson & Clarke, 2004); however, the awareness

dimension is mostly observed among the students who have deficiencies in problem-solving skills (Kuzle, 2018). When the number of cognitive behaviours demonstrated by the groups was taken into consideration, it was found that there was a significant difference between the total number of cognitive behaviours of the students in the outstanding achievement group and the number of cognitive behaviours of other groups. The total number of cognitive behaviours demonstrated by the students with outstanding achievement levels was about twice more when they are compared with those with high achievement levels; besides, it was approximately three times more, compared with the total number of cognitive behaviours demonstrated by the students with moderate achievement and seven times more when they are compared with the total number of cognitive behaviours demonstrated by the students with low achievement level. In addition, as there was a significant difference in the total number of cognitive behaviours demonstrated between the students with outstanding achievement level and the total number of behaviours of the students with moderate achievement level, it was found that there was a significant difference in terms of the number of cognitive behaviours between all the groups and the student group with low achievement level. Considering these results, it can be stated that good problem solvers make more operations and take notes during the problem-solving process. The result suggesting that the behaviours in the evaluation are applied most during the process of problem-solving is parallel with the results reached by Wilson and Clarke (2004) and Wong (1992) in different class levels. In addition, the results suggest that the solution of the problem is a complex process consisting of continuous change between metacognition and cognition as it is referred to in the relevant literature (Artzt & Armour-Thomas, 1992; Author, 2016; Barbacena & Sy, 2015; Dunlosky, 1998; Kuzle, 2018, 2019; Lester, Garofalo & Kroll, 1989; Schoenfeld, 1992; Wilson & Clarke, 2004).

When the problem-solving durations of the groups were analysed, it was seen that there was a decrease in duration from the student group with outstanding achievement levels to the students with low achievement levels. The total amount of time that the students with outstanding achievement level spent during the problem-solving process are twice more than the total time spent by the students with low achievement level. In addition, the total time that the students in the group of outstanding achievement spent during the problem-solving process are approximately twice more than the total amount of time spent by the students with moderate achievement in problem-solving. As the total amount of time spent by the students with outstanding achievement and high achievement in problem-solving is taken into consideration, it is seen that it is a difference that may be regarded as significant for the problem-solving time. When the problem-solving time for all of the groups was taken into account, it was concluded that it was a difference that should be regarded as significant for the problem-solving process.

As the time spent by the group members, who started the problem-solving process with reading the problem first, was analysed, it was realised that the students with

outstanding achievement spent less time in their first reading the problem; on the other hand, the students with low achievement spent more time in their first reading the problem. This situation has an inverse correlation with the total amount of time that the group spent solving a problem. While the total amount of time that the groups spent in the problem-solving process decreased from the students with outstanding achievement to the students with low achievement level, the total amount of time that the groups spent in their first reading the problem increased from the students with outstanding achievement level to those with low achievement level. When this situation was analysed, it was concluded that the students with outstanding achievement levels spent the time left from reading the problem to problem-solving by reading the question faster than the other groups. It can be claimed that this situation is the same from the group consisting of the students with outstanding achievement to those with low achievement. In this context, the students with outstanding achievement spent more time in the problem-solving process by reading the question faster than the students with high, moderate and low achievement, the students with high achievement than the students with moderate and low achievement, the students with moderate achievement than those with low achievement.

The result, suggesting that the time spent in the problem-solving process decreases from the students with outstanding achievement to those with low achievement, overlaps with the result reached by Pressley (1995) and Şahin (2007). Pressley (1995) and Şahin (2007) found that successful problem solvers analyse the problem for a long time before starting to solve the problem; they assess whether they make progress in the problem or not. If not, they change their strategies by returning at the beginning, that the students with low achievement do not spend any time analysing and planning during the problem-solving process. In addition, Hoy (2004) claimed that the students, with high management of academic work or task, try alternative strategies, that they obstinately continue their work until they reach a result by making more effort to be successful. These situations indicate that successful students spend more time using more strategies in the problem-solving process.

Successful students struggle for the mental representation of the problem that includes shapes (Montague, Applegate, & Marquard, 1993). The problem solvers with low achievement levels work passively in the problem-solving process by trying direct routes to the solution and some difficulties they encounter distract them easily (Breslow, 2001). Successful problem-solvers work actively in the problem-solving process; try to find alternative ways of the solution by checking their solutions (Cai & Brook, 2006). Successful problem solvers follow their progress actively in the problem-solving process, deciding which strategies or approaches to pursue, or which ones to change and abandon (Seldon & Seldon, 1997). Individuals with highly developed metacognitive skills make decisions in solving the problem by selecting appropriate strategies; achieving successful actions overcoming the problem and spending time to think about their processes

(NCREL, 1995). It was noticed that the results gathered from the successful problem solvers who spend time thinking in the problem-solving process, interest in figures for the mental representation of the problem, work actively in the problem-solving process, try different methods and monitor their progress. The results from the problem solvers with low achievement who are passive in the problem-solving process and easily distracted by the difficulties they encounter overlaps with the results reached within the scope of this study.

When the behaviour dimension that the groups started the process was considered, it was concluded that the groups started the process with the behaviours in the awareness dimension most and they mostly completed the process with the behaviours in the evaluation dimension. Similar results were reported by (Author, 2016; Barbacena & Sy, 2015; Fazira et al., 2020; Kuzle, 2018, 2019; Wilson & Clarke, 2004). The behaviour that was demonstrated least in each achievement group was in the awareness dimension. This situation was reported by Wilson and Clarke (2004) by attributing the low frequency of reporting to the fact that mindfulness is not recognised by the student as a metacognitive activity due to its less distinctive character (in the sense of conscious calling to mind) compared to evaluation and regulation. In addition, Baş (2016) reported the research results that student either demonstrated or did not demonstrate metacognitive behaviours without being aware of them. As the behaviour order is considered in general, it can be stated that the students demonstrated the behaviours in the dimension of awareness at the beginning of the problem-solving process, behaviours in the dimension of regulation in the middle and behaviours in the dimensions of cognitive and evaluation through the end of the process most.

There are some recommendations below depending on the results reached in the research.

• It can be beneficial in reaching a meaningful solution during problem-solving when the teacher guide students by focusing on some behaviours respectively related to the dimensions of awareness, regulation and evaluation during solving a problem in class, showing the way to the students in the most demonstrated behaviours in all dimensions.

• As the students demonstrate behaviours related to the evaluation dimension most in number within the scope of awareness, regulation and evaluation dimensions in the problem-solving process, research to be conducted only on the behaviours relevant to this dimension can contribute to the problem-solving process.

• As good problem solvers generally take notes of what is given in the problem, teachers may emphasise this point in the problem-solving process. It can be ensured that the students do regular exercises to solve the problem and take notes on paper.

• As this research was limited to the eighth-class students, further research can be conducted on a larger sample to increase the generalisability of the gathered findings.

• As only one question was used in this research, more than one problem sentence can be used in future research to increase the generalisability of the similar and different behaviours that students demonstrate in the problem-solving process.

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Ethics Approval: Permission for this research was obtained from the Human Research Ethics Committee of Erzincan Binali Yıldırım University with the protocol number 05/08.

References

Abdullah, A. H., Rahman, S. N. S. A., & Hamzah, M. H. (2017). Metacognitive skills of Malaysian students in non-routine mathematical problem-solving. *Bolema, Rio Claro (SP), 31*(57), 310-322. **http://doi.org/10.1590/1980-4415v31n57a15**

Abu-Elwan, R. (1999). The development of mathematical problem posing skills for prospective middle school teachers. In A. Rogerson (Ed.), *Proceedings of the International conference on mathematical education into the 21st century: Social challenges, issues and approaches* (Vol. II, pp. 1–8). Cairo, Egypt.

Adibnia, A., & Putt, I. J. (1998). Teaching problem solving to year 6 students: A new approach. *Mathematics Education Research Journal, 10*(3), 42-58.

Ağaç, G., & Masal, E. (2017). The relationship between 8th-grade students' opinions about problem-solving, beliefs about mathematics, learned hopelessness and academics success. *Gaziantep University Journal of Social Sciences, 16*(1), 216-229. **http://doi.org/10.21547/jss.282104**

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem-solving in small groups, *Cognition and Instruction, 9*(2), 137-175.

Baş, F. (2016). Prospective secondary mathematics teachers' metacognitive awareness and metacognitive behaviours in problem solving processes. *Universal Journal of Educational Research, 4*(4), 779-801. **http://doi.org/10.13189/ujer.2016.040416**

Aydemir, H., & Kubanç, Y. (2014). Investigation of the cognitive behavioral problem solving process. *Turkish Studies, 9*(2), 203-219. **http://doi.org/10.7827/TurkishStudies.6555**

Barbacena, L. B. & Sy, N. R. (2015). Metacognitive model in mathematical problem solving. *Intersection, 12*(1), 16-22.

Bhat, Ö. A. (2014). Effect of problem solving ability on the achievement in mathematics of high school students. *Indian Journal of Applied Research, 4*(8), 685-688.

Breslow, L. (2001). *Transforming novice problem solvers into experts.* Online available from http://web.mit.edu/fnl/vol/133/breslow.htm

Brown, A. L. (1978). Knowing when, where, and how to remember: A problem of metacognition. In R. Glaser (Ed.), *Advances in instructional psychology*, Vol. 1 (pp. 77–165). Hillsdale: Erlbaum.

Cai, J., & Brook, M. (2006). Looking back in problem solving. *Mathematics Teaching Incorporating Micromath, 196*, 42–45.

Cooper, S. E., & Robinson, D. A. G. (1991). The relationship of mathematics self-efficacy beliefs to mathematics anxiety and performances. *Measurement and Evaluation in Counseling and Development, 24*(1), 4-11.

Coutinho, S. A. (2007). The relationship between goals, metacognition and academic success. *Educate,* 7(1), 39-47.

Creswell, J. W. (2007). *Qualitative inquiry and research design choosing among five traditions,* Thousand Oaks: Sage Publications, Inc.

Demir, Ö. (2013). An investigation of the prospective teachers level of usıng cognitive awareness skills while studying: A qualitative study. *H. U. Journal of Education, 44*, 133-148.

Demir, Ö. D. (2016) An investigation of secondary school students' levels of problem solving and metacognitive skills, *Kastamonu Education Journal, 24*(2), 789-802.

Desoete, A., Roeyers, H., & De Clercq, A. (2003). Can offline metacognition enhance mathematical problem solving, *Journal of Educational Psychology, 95*(1), 188-200. **http://doi.org/10.1037/0022-0663.95.1.188**

Doğanay, A., & Demir, Ö. (2011) Comparison of the level of using metacognitive strategies during study between high achieving and low achieving prospective teachers. *Educational Sciences: Theory & Practice, 11*(4), 2021-2043.

Fazira, S. K., Sukoriyanto, & Rahardjo, S. (2021, March). Identification of young students' metacognition in mathematical problem solving using the multi-method interview approach. In *AIP Conference Proceedings* (Vol. 2330, No. 1, p. 040012). AIP Publishing LLC.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. L. Resnick (Ed.), *The nature of intelligence içinde* (s. 231-236). Hillsdale, NJ: Erlbaum.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal of Research in Mathematics Education, 16*, 163–176.

Hackett, G., & Betz, N.E. (1989). An exploration of the mathematics self efficacy mathematics performance correspondence. *Journal for Research in Mathematics Education, 20*(3), 261-273. **http://doi.org/10.2307/749515**

Hoy, A. W. (2004, April). *What do teachers know about self-efficacy*. Paper presented at the Annual Meeting of the American Educational Research Association, San Diego, CA.Institute and State University, Blacksburg, Virginia.

Işık, C., & Kar, T. (2011) Investigating of the number perception and nonroutine problem solving skills of 6th, 7th and 8th grade students, *Ahi Evran University Journal of Kırşehir Education Faculty, 12*(1), 57-72.

Jaafar, W. M. W., & Ayub, A. F. M. (2010). Mathematics self-efficacy and meta-cognition among university students, *Procedia Social and Behavioral Sciences, 8*, 519–524. **http://doi.org/10.1016/j.sbspro.2010.12.071**

Kanadlı, S., & Sağlam, Y. (2013) Is metacognitive strategies effective ın problem solving. *Elementary Education Online, 12*(4), 1074-1085.

Kaplan, A., Duran, M., & Baş, G. (2016). Examination with the structural equation modeling of the relationship between mathematical metacognition awareness with skill perception of problem solving of secondary school students. *Inonu University Journal of the Faculty of Education, 17*(1), 1-16. **http://doi.org/10.17679/iuefd.17119785**

Kaur, B. (1997). Difficulties with problem solving in mathematics. *The Mathematics Educator, 2*(1), 93-112.

Kazemi, F., Fadae, M. R., & Bayat, S. (2010). A subtle view to metacognitive aspect of mathematical problems solving. *Procedia Social and Behavioral Sciences, 8*, 420- 426.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks, Educational Studies in Mathematics, 49(2), 225-250. **http://doi.org/10.1023/A:1016282811724**

Kuzle, A. (2018). Assessing metacognition of grade 2 and grade 4 students using an adaptation of multi-method interview approach during mathematics problem-solving. *Math Ed Res J, 30*, 185–207. **http://doi.org/10.1007/s13394-017-0227-1**

Kuzle, A. (2019). Second graders' metacognitive actions in problem solving revealed through action cards. *The Mathematics Educator, 28*(1), 27-60.

Lesh, R. (1985). Conceptual analysis of problem-solving performance. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 309-329), Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. K. (1980) Mathematical problem solving research. In R. J. Shumway (Ed.), *Research in mathematics education* (pp. 286-323). Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K. (1985). Methodological considerations in research on mathematical problem-solving instruction. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 41—69), Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K. , Garofalo, J., & Kroll, D. L. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classes*. Final report. Online available from ERIC (ED314255).

Lester, F. K. J., Garofalo, J., & Kroll, D. L. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classe*s. Online available from ERIC (ED314255).

Lester, F. K., & Kehle, P.E. (2003). From problem solving to modeling: The Evolution of thinking about reseach on complex mathematical activity. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning and teaching* (pp. 501-517). Mahwah: Lawrance Erlbaum Associates Publishers.

Mayer, R. (1998). Cognitive, metacognitive, and motivational aspects of problem solving. *Instructional Science, 26*(1), 49-63. **http://doi.org/10.1023/A:1003088013286**

McGinn, M. K., & Boote, D. N. (2003). A first-person perspective on problem solving in a history of mathematics course. *Mathematical Thinking and Learning, 5*(1), 71-107. doi: 10.1207/S15327833MTL0501_03

Merriam, S. B., & Grenier, R. S. (2019). *Qualitative research in practice: Examples for discussion and analysis*. San Francisco, CA: Jossey-Bass Publishers.

Mevarech, Z. R. (1999). Effects of metacognitive training embedded in cooperative settings on mathematical problem solving, *The Journal of Educational Research, 92*(4), 195-205. doi: 10.1080/00220679909597597

Mevarech, Z. R., & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning, *British Journal of Educational Psychology,73*(4), 449–471. **http://doi.org/10.1348/000709903322591181**

Mokos, E., & Kafoussi, S. (2013). Elementary students' spontaneous metacognitive functions in different types of mathematical problems. *REDIMAT - Journal of Research Math. Educ., 2*(2), 242–267. **http://doi.org/10.4471/redimat.2013.29**

Montague, M., Applegate, B., & Marquard, K. (1993). Cognitive strategy instruction and mathematical problem-solving performance of students with learning disabilitie. *Learning Disabilities Research and Practice, 29*, 251-261.

Nancarrow, M. (2004). *Exploration of metacognition and non-routine problem based mathematics instruction on undergraduate student problem solving success* (Unpublished Doctoral Dissertation). The Florida State University College of Education, Florida.

National Council of Teachers of Mathematics [NCTM], (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics [NCTM], (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics [NCTM], (1995). *Assessment standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics [NCTM], (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

North Central Regional Educational Laboratory [NCREL] (1995). *Strategic teaching and reading project guidebook*. Online available from http://www.ncrel.org/sdrs/areas/issues/students/learning/lr1metp.htm.

Ormrod, J. E. (2003). *Educational psychology*. New Jersey: Merill Prentice Hall.

Özgen, K., Ay, M., Kılıç, Z., Özsoy, G., & Alpay, F. N. (2017). Investigation of middle school students' learning styles and attitudes towards mathematical problem solving, *Mehmet Akif Ersoy University Journal of Education Faculty, 41*, 215-244. **http://doi.org/10.21764/efd.55023**

Özsoy, G. (2005). The relationship between problem solving skills and mathematical achievement. *Gazi University Journal of Gazi Educational Faculty, 25*(3), 179-190.

Özsoy, G. (2011). An investigation of the relationship between metacognition and mathematics achievement. *Asia Pacific Educ. Rev., 12*(2), 227-235. doi: 10.1007/s12564-010-9129-6

Özsoy, G., & Ataman, A. (2009). The effect of metacognitive strategy trainingon mathematical problem solving achievement, *International Electronic Journal of Elementary Education, 1*(2), 68-83.

Panaoura, A., Philippou, G., & C. Christou, C. (2003). *Young pupils' metacognitive ability in mathematics.* Paper presented at CERME 3: Third Conference of the European Society for Research in Mathematics Education. Online available from http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG3/TG3_Panaoura_cerme3.pdf,

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. Beverly Hills, CA: Sage

Pintrich, P. R., & De Groot, E. (1990). Motivation aland self regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*(1), 33-40.

Polya, G., 1973. *How to solve it? A new aspect of mathematical method* (2 nd Ed.). New Jersey: Princeton University Press.

Pressley, M. (1995). *Cognition, teaching, assesment*. New York: Harper Collins College Publishers.

Saumure, K., & Given Lisa M. (2008). *The SAGE encyclopedia of qualitative research methods*. Thousand Oaks, CA: Sage.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics, In D. A. Grows (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schunk, D. H., & Zimmerman, B. J. (1998). *Self- regulated learning from teaching to self reflective practice*, New York: Guilford Press.

Seldon, A., & Seldon, J. (1997). *What does it take to be an expert problem solver.* Online available from https://www.academia.edu/2313711/What_Does_it_Take_to_be_an_Expert_Problem_Solver_September_1997?auto=download.

Senemoğlu, N. (2007). *Gelişim, öğrenme ve öğretim [Development, learning and teaching]*. Ankara: Gönül Publishing.

Silver, E. (1985). *The teaching and assessing of mathematical problem solving*. Reston, VA: National Council of Teachers of Mathematics [NCTM].

Sweeney, C. M. (2010). *The metacognitive functioning of middle school students with and without learning disabilities during mathematical problem solving*. Open Access Dissertation, University of Miami.

Şahin, A. A. (2007). Determining the problem solving strategies for the students aged 13-14 years. (Unpublished master's thesis). Balıkesir University Institute of Science, Balıkesir.

Şengül, S. & F. Yıldız, F. (2013). The relation between the metacognitive behaviours demonstrated by students in their problem solving process with collaborative learning groups and their mathematical self- efficacy, *International Journal of Social Science, 6*(1), 1295-1324. **http://doi.org/10.9761/JASSS_400**

Şengül, S., & Y. Katrancı, Y. (2015). Meta-cognitive aspects of solving indefinite integral problems, *Procedia - Social and Behavioral Sciences, 197*, 622 – 629. **http://doi.org/10.1016/j.sbspro.2015.07.205**

Tüysüz, C. (2013). Determination of gifted students' metacognition level about problem solving skills, *Mustafa Kemal University Journal of Social Sciences Institute, 10*(21), 157-166.

Uysal, O. (2007*). Evaluation of relation between problem solving skills, anxiety and attitudes of secondary education grade ii students directed towards mathematics course* (Unpublished master's thesis). Dokuz Eylül University Institute of Educational Sciences, İzmir.

Veenman, M. V. J., Van Hout-Wolters, B. H. A. M., & Afflerbach, P. (2006). Metacognition and learning: Conceptual and methodological considerations. *Metacognition Learning, 1*, 3–14.

Verschaffel, L. (1999). Realistic mathemati-cal modelling and problem solving in theupper elementary school: Analysis andimprovement. In J. H. M. Hamers, J. E. H.Van Luit, & B. Csapo (Eds.)*, Teaching andlearning thinking skills: Contexts of learning* (pp. 215-240). Lisse, The Netherlands:Swets & Zeitlinger.

Verschaffel L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999) Learning to solve mathematical application problems: A design experiment with fifth graders, *Mathematical Thinking and Learning, 1*(3), 195-229. **http://doi.org/10.1207/s15327833mtl0103_2**

Wilson, J. (2001). Methodological difficulties of assessing metacognition: A new approach. Retrieved from (10.08.2018): http://eric.ed.gov/pdfs/ed460143.pdf

Wilson, J., & Clarke, D. (2004). Towards the modelling of mathematical metacognition, *Mathematics Education Research Journal, 16*(2), 25-48.

Wong, P. (1992). Metacognition in mathematical problem solving. *Singapore Journal of Education, 12*(2), 48-58. **http://doi.org/10.1080/02188799208547691**

Yimer, A., & N. F. Ellerton. (2010) A five-phase model for mathematical problem solving: Identifying synergies in pre-service-teachers' metacognitive and cognitive actions. *ZDM Mathematics Education, 4*2(2), 245–261. **http://doi.org/10.1007/s11858-009-0223-3**

Yong, . T. Y., & Kiong. L. N. K. (2006). *Metacognitive aspect of mathematics problem solving*. MARA University of Technology Malaysia, Kuala Lumpur.

Young, A., & Fry, J. D. (2008). Metacognitive awareness and academic achievement in college students. *Journal of the Scholarship of Teaching and Learning, 8*(2), 1-10.

Zawojewski, J.S. ve Lesh, R. (2003). A models and modeling perspective on problem solving. In R. Lesh & H. Doerr, (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 317-337). Mahwah, NJ: Erlbaum.